

# Year 7 – Algebraic thinking...



# Algebraic notation

## What do I need to be able to do?

By the end of this unit you should be able to:

- Be able to use inverse operations and "operation families".
- Be able to substitute into single and two step function machines.
- Find functions from expressions.
- Form sequences from expressions.
- Represent functions graphically.

## Keywords

**Function:** a relationship that instructs how to get from an input to an output.

**Input:** the number/ symbol put into a function.

**Output:** the number/ expression that comes out of a function.

**Operation:** a mathematical process

**Inverse:** the operation that undoes what was done by the previous operation. (The opposite operation)

**Commutative:** the order of the operations do not matter.

**Substitute:** replace one variable with a number or new variable.

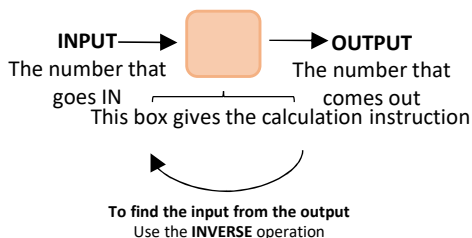
**Expression:** a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

**Evaluate:** work out

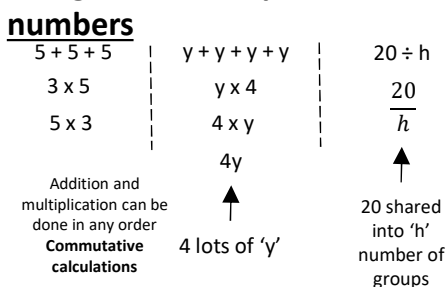
**Linear:** the difference between terms increases or decreases by the same value each time

**Sequence:** items or numbers put in a pre-decided order

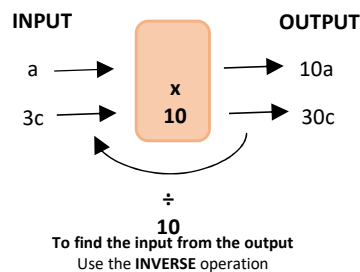
## Single function machines



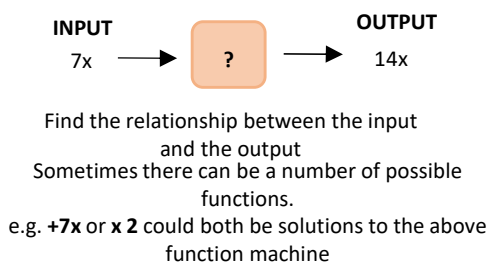
## Using letters to represent numbers



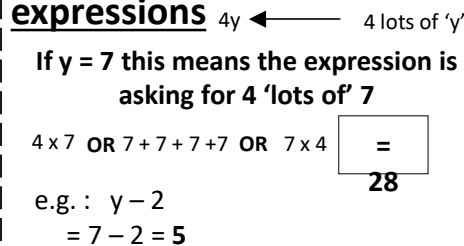
## Single function machines (algebra)



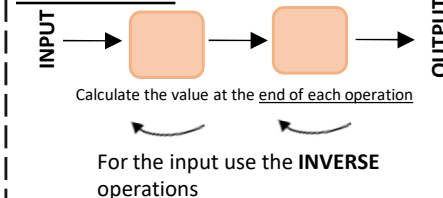
## Find functions from expressions



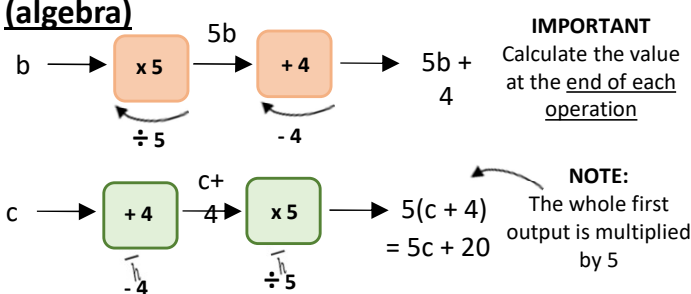
## Substitution into expressions



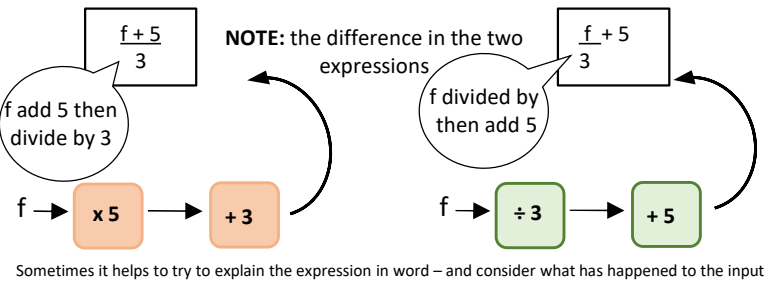
## Two step function machines



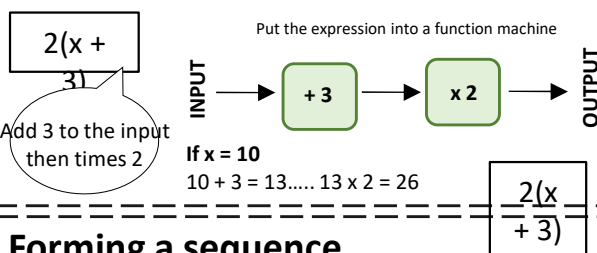
## Two step function machines (algebra)



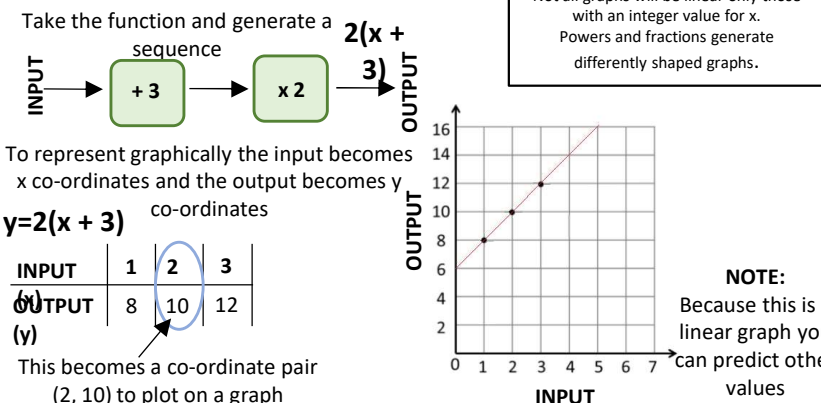
## Find functions from expressions



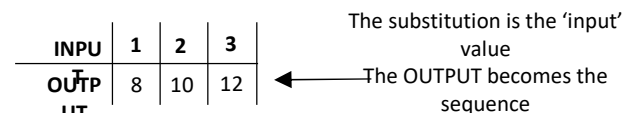
## Substitution into an expression



## Representing functions graphically



## Forming a sequence



<p><b>Question 1</b></p> <p>What is the inverse of addition?</p>	<p><b>Question 2</b></p> <p>If <math>x=10</math> , what is the value of <math>3x</math> ?</p>	<p><b>Question 3</b></p> <p>Simplify the expression <math>y + y + y</math></p>	<p><b>Question 4</b></p> <p>What is the inverse of a cube root?</p>
<p><b>Question 5</b></p> <p>Simplify the expression <math>k \div 2</math></p>	<p><b>Question 6</b></p> <p>True or false: <math>a \div b = \frac{a}{b}</math></p>	<p><b>Question 7</b></p> <p>If <math>x = 20</math>, what is <math>2x + 5</math> ?</p>	<p><b>Question 8</b></p> <p>What is the inverse of multiplication?</p>
<p><b>Question 9</b></p> <p>If <math>x=4</math>, what is the value of <math>10 - x</math> ?</p>	<p><b>Question 10</b></p> <p>What is the inverse of subtraction?</p>	<p><b>Question 11</b></p> <p>Simplify the expression <math>p \times 10</math></p>	<p><b>Question 12</b></p> <p>If <math>x = 2.5</math>, what is <math>3x - 4</math> ?</p>
<p><b>Question 13</b></p> <p>What is the inverse of squaring?</p>	<p><b>Question 14</b></p> <p>Simplify the expression <math>z \times z</math></p>	<p><b>Question 15</b></p> <p>If <math>x=1.2</math>, what is <math>4x + 10</math> ?</p>	<p><b>Question 16</b></p> <p>True or false: if <math>a=0.5</math> and <math>b=5</math>, <math>\frac{a}{b}</math> is greater than <math>\frac{b}{a}</math></p>
<p><b>Question 17</b></p> <p>True or false: Input = <math>2x</math> Function : Square Output = <math>2x^2</math></p>	<p><b>Question 18</b></p> <p>If <math>x=5</math>, what is the value of <math>7+x</math> ?</p>	<p><b>Question 19</b></p> <p>What is the inverse of division?</p>	<p><b>Question 20</b></p> <p>Simplify the expression <math>2 \times a \times 3</math></p>

Answers: 1.Subtraction 2. 30 3.  $3y$  4. Cubing 5.  $\frac{k}{2}$  6. True 7. 45 8. Division 9. 6 10. Addition 11.  $10p$  12. 3.5 13. Square rooting 14.  $z^2$  15. 14.8 16. True 17. False 18. 12 19. Multiplication 20.  $6a$



# Year 7 – Algebraic thinking...

## Equality and Equivalence

### What do I need to be able to do?

By the end of this unit you should be able to:

- Form and solve linear equations
- Understand like and unlike terms
- Simplify algebraic expressions

### Keywords

- Equality:** two expressions that have the same value  
**Equation:** a mathematical statement that two things are equal  
**Equals:** represented by '=' symbol – means the same  
**Solution:** the set or value that satisfies the equation  
**Solve:** to find the solution.  
**Inverse:** the operation that undoes what was done by the previous operation. (The opposite operation)  
**Term:** a single number or variable  
**Like:** variables that are the same are 'like'  
**Coefficient:** a multiplicative factor in front of a variable e.g.  $5x$  (5 is the coefficient,  $x$  is the variable)  
**Expression:** a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

### Equality

$$2 + 14 = 5 + 5 + 6$$

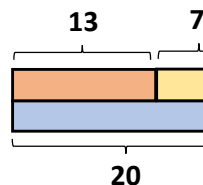
16      16  
"Is equal to"

Saying it out loud sometimes helps you to understand equality

The sum on the left has the same result as the sum on the right

### Fact Families

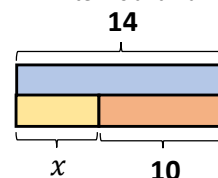
Use a bar model to display the relationships between terms and numbers.



$$13 + 7 = 20 \quad 20 - 7 = 13$$

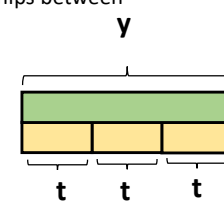
$$7 + 13 = 20 \quad 20 - 13 = 7$$

Model the information



$$x + 10 = 14 \quad 14 - 10 = x$$

$$10 + x = 14 \quad 14 - x = 10$$



$$t + t + t = y \quad y - t - t = t$$

$$3t = y \quad y \div 3 = t$$

$$3t = y \quad y \div t = 3$$

### Solve one step equations (+/-)

There is more to this than just spotting the answer

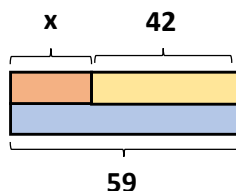
$$x + 42 = 59$$

$$x + 42 = 59$$

$$42 + x = 59$$

$$59 - x = 42$$

$$59 - 42 = x$$



Don't forget you know how to use function machines



### Solve one step equations (x/÷)

$$\frac{f}{4} = 5$$

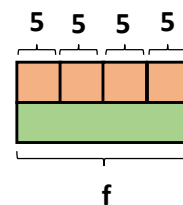
$$\frac{f}{4} = 5$$

$$f \div 4 = 5$$

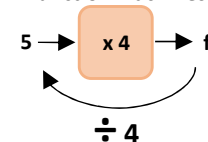
$$f \div 5 = 4$$

$$5 \times 4 = f$$

$$4 \times 5 = f$$



Don't forget you know how to use function machines



### Like and unlike terms

Like terms are those whose variables are the same

♥ and 3♥ are like terms

the variable is the same  
★ and 3♥ are unlike terms

the variables are NOT the same

Examples and non-examples

#### Like terms

$y, 7y$   
 $2x^2, x^2$   
 $ab, 10ba$   
 $5, -2$

#### Un-like terms

$y, 7x$   
 $2x^2, 2c^2$   
 $ab, 10a$   
 $5, -2t$

Note here  $ab$  and  $ba$  are commutative operations, so are still like terms

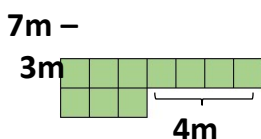
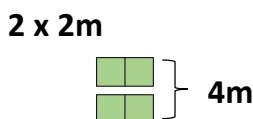
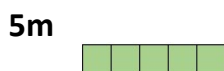
### Equivalence

Check equivalence by substitution  
e.g.  $m=10$

<b>5m</b>	<b>2x</b>	<b>7m -</b>
$5 \times 10$	$2 \times (2 \times 10)$	$7 \times 10 -$
$10 \times 5$	$= 2 \times 20$	$(3 \times 10) -$
$= 50$	$= 40$	$= 70 - 30$
		$= 40$

Equivalent expressions

Repeat this with various values for  $m$  to check



### Collecting like terms = symbol

The = symbol means equivalent to.

It is used to identify equivalent expressions

#### Collecting like terms

Only like terms can be combined

$$4x + 5b - 2x + 10b$$

$$2x + 15b$$

#### Common misconceptions

$$2x + 3x^2 + 4x \equiv 6x + 3x^2$$

Although they both have the  $x$  variable  $x^2$  and  $x$  terms are un-like terms so can not be collected

<b>Question 1</b> Simplify $3a - a + 4a - 3a$	<b>Question 2</b> Simplify $8a - a - 2a + 4a$	<b>Question 3</b> Solve $x - 10 = -8$	<b>Question 4</b> Solve $x - 10 + -6$
<b>Question 5</b> Simplify $7a + a + 3a + 3a$	<b>Question 6</b> Simplify $4a + 3a - 4a + 5a$	<b>Question 7</b> Solve $9 - x = 2$	<b>Question 8</b> Solve $13 - x = 10$
<b>Question 9</b> Simplify $4a + 4a + 4a - 3a$	<b>Question 10</b> Simplify $6x - 4x + 3x$	<b>Question 11</b> Solve $8x = 80$	<b>Question 12</b> Solve $\frac{x}{8} = 4$
<b>Question 13</b> Simplify $3x + 5x - 3x + 4x$	<b>Question 14</b> Simplify $5a - 2a - a - 2a$	<b>Question 15</b> Solve $x + 4 = 8$	<b>Question 16</b> Solve $x + 4 = 11$
<b>Question 17</b> Simplify $6a - a + 3a - 2a$	<b>Question 18</b> Simplify $8a + a + 3a + 2a$	<b>Question 19</b> Solve $x - 10 = 0$	<b>Question 20</b> Solve $2x = 8$

Answers 1.3a, 2. 9a, 3. X=2, 4. X=4, 5. 14a, 6. 8a, 7. X=7, 8. X=3, 9. 9a, 10.5x, 11. X=10, 12. X=32, 13. 9x,14. 0, 15. X=4, 16. X=7, 17. 6a, 18. 14a, 19.x=10, 20. X=4

# Year 7 – Algebraic thinking...



# Sequences

## What do I need to be able to do?

By the end of this unit you should be able to:

- Describe and continue both linear and non-linear sequences
- Explain term to term rules for linear sequence
- Find missing terms in a linear sequence

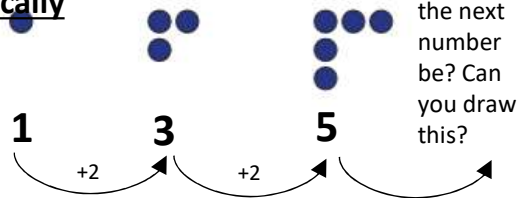
## Keywords

- Sequence:** items or numbers put in a pre-decided order
- Term:** a single number or variable
- Position:** the place something is located
- Rule:** instructions that relate two variables
- Linear:** the difference between terms increases or decreases by the same value each time
- Non-linear:** the difference between terms increases or decreases in different amounts
- Difference:** the gap between two terms
- Arithmetic:** a sequence where the difference between the terms is constant
- Geometric:** a sequence where each term is found by multiplying the previous one by a fixed non zero number

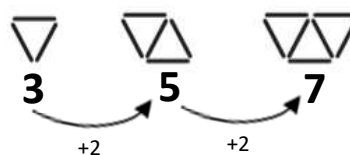


## Describe and continue a sequence diagrammatically

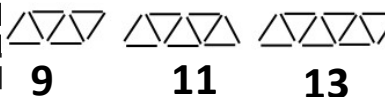
Count the number of circles or lines in each image



## Predict and check terms



CHECK – draw the next terms



### Predictions:

Look at your pattern and consider how it will increase.

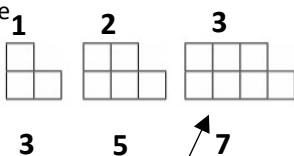
e.g. How many lines in pattern 6?

**Prediction - 13**

If it is increasing by 2 each time - in 3 more patterns there will be 6 more lines

## Sequence in a table and graphically

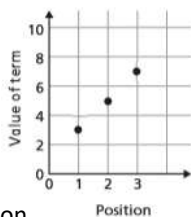
**Position:** the place in the sequence



**Term:** the number or variable (the number of squares in each image)

Position	1	2	3
Term	3	5	7

### Graphically



Because the terms increase by the same addition each time this is **linear** – as seen in the graph

## Linear and Non Linear Sequences

**Linear Sequences** – increase by addition or subtraction and the same amount each time

**Non-linear Sequences** – do not increase by a constant amount – quadratic, geometric and Fibonacci.

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division.

**Fibonacci Sequence** – look out for this type of sequence

0 1 1 2 3 5 8 ...

Each term is the sum of the previous two terms.

## Continue Linear Sequences

7, 11, 15, 19...

**How do I know this is a linear sequence?**

It increases by adding 4 to each term.

**How many terms do I need to make this conclusion?**

At least 4 terms – two terms only shows one difference not if this difference is constant. (a common difference).

**How do I continue the sequence?**

You continue to repeat the same difference through the next positions in the sequence.



## Continue non-linear Sequences

1, 2, 4, 8, 16 ...

**How do I know this is a non-linear sequence?**

It increases by multiplying the previous term by 2. – this is a geometric sequence because the constant is multiply by 2

**How many terms do I need to make this conclusion?**

At least 4 terms – two terms only shows one difference not if this difference is constant. (a common difference).

**How do I continue the sequence?**

You continue to repeat the same difference through the next positions in the sequence.



## Explain term-to-term rule

How you get from term to term  
 Try to explain this in full sentences not just with mathematical notation.  
 Use key maths language – doubles, halves, multiply by two, add four to the previous term etc.

To explain a whole sequence you need to include a term to begin at...

The next term is found by tripling the previous term. The sequence begins at 4.

4, 12, 36, 108...

First term

<p><b>Question 1</b></p> <p>Find the missing numbers 31,_,21_,11,6</p>	<p><b>Question 2</b></p> <p>Find the missing numbers 35,_,25_,15,10</p>	<p><b>Question 3</b></p> <p>Find the next 2 terms 1,3,4,7</p>	<p><b>Question 4</b></p> <p>Find the next 2 terms 81,27,9</p>
<p><b>Question 5</b></p> <p>Which of the equations will produce a straight line graph <math>Y = \frac{1}{x} + 2</math> or <math>y = x + 2</math>?</p>	<p><b>Question 6</b></p> <p>Find the missing numbers 29,_,19,14,_,14,_,4</p>	<p><b>Question 7</b></p> <p>Find the missing numbers 55,47,_,31,_,15</p>	<p><b>Question 8</b></p> <p>Find the next 2 terms 1,2,3,5</p>
<p><b>Question 9</b></p> <p>Find the next 2 terms 256,64,16</p>	<p><b>Question 10</b></p> <p>Which of the equations will produce a straight line graph <math>Y = 10x + 8</math> or <math>Y = \frac{8}{x+10}</math> ?</p>	<p><b>Question 11</b></p> <p>Find the missing numbers 10,15,_,25,_,35</p>	<p><b>Question 12</b></p> <p>Find the missing numbers _,17,23,29,_,41</p>
<p><b>Question 13</b></p> <p>Find the next 2 terms 81,27,9</p>	<p><b>Question 14</b></p> <p>Find the next 2 terms 10,50,250</p>	<p><b>Question 15</b></p> <p>Which of the equations will produce a straight line graph <math>y = 3x + 2</math> or <math>Y = \frac{3}{x} + 2</math> ?</p>	<p><b>Question 16</b></p> <p>Find the missing numbers _,49,39,29,19,_,</p>
<p><b>Question 17</b></p> <p>Find the missing numbers 3,_,15,21,_,33</p>	<p><b>Question 18</b></p> <p>Find the next 2 terms 5,2,7,9</p>	<p><b>Question 19</b></p> <p>Find the next 2 terms 81,27,9</p>	<p><b>Question 20</b></p> <p>Find the next 2 terms 30000,3000,300</p>

Answers: 1.26,16,2.30,20,3.11,18,4.3,1,5.y+ x + 2,6. 24,9, 7. 39,23 8.8,13, 9.4,1, 10.y=10x + 8, 11.20,30, 12.11,35, 13.3,1, 14. 1250,6250, 15.  $Y = 3x + 2$ , 16. 59,9, 17.9,27, 18.16,25, 19.3,1, 20.30,3

## FDP equivalence

### What do I need to be able to do?

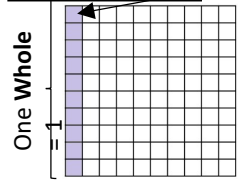
By the end of this unit you should be able to:

- Convert fluently between fractions, decimals & percentages

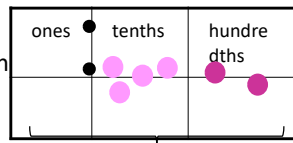
### Keywords

- Fraction:** how many parts of a whole we have
- Decimal:** a number with a decimal point used to separate ones, tenths, hundredths etc.
- Percentage:** a proportion of a whole represented as a number between 0 and 100
- Place value:** the numerical value that a digit has decided by its position in the number
- Placeholder:** a number that occupies a position to give value
- Interval:** a range between two numbers
- Tenth:** one whole split into 10 equal parts
- Hundredth:** one whole split into 100 equal parts
- Sector:** a part of a circle between two radius (often referred to as looking like a piece of pie)
- Recurring:** a decimal that repeats in a given pattern

### Tenths and hundredths



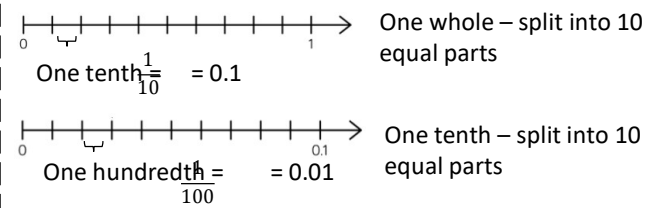
One hundredth (one whole split into 100 equal parts) =  $\frac{1}{100} = 0.01$



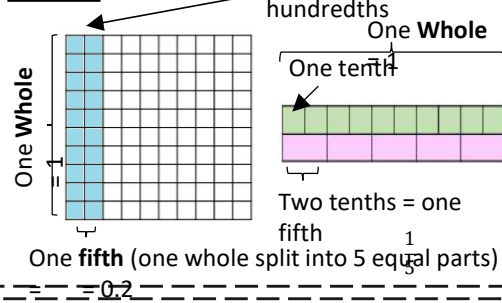
0 ones, 5 tenths and 2 hundredths  
 $0 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.01 + 0.01$   
 $0.01$   
 $= 0 + 0.5 + 0.02$   
 $= 0.52$

One tenth (one whole split into 10 equal parts) =  $\frac{1}{10} = 0.1$

### On a number line

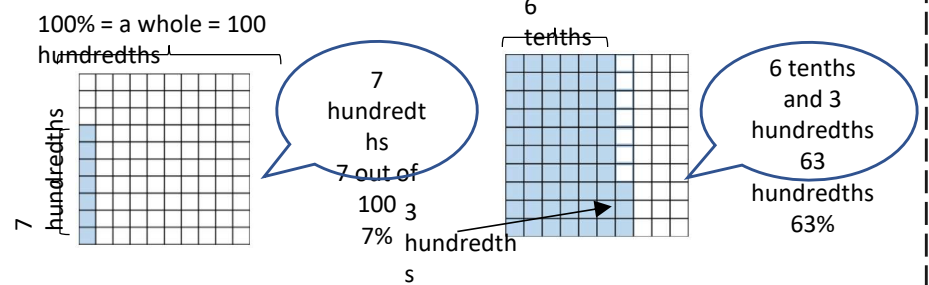


### Fifths



Twenty hundredths  
 One Whole  
 One tenth  
 Two tenths = one fifth  
 One fifth (one whole split into 5 equal parts) =  $\frac{1}{5} = 0.2$

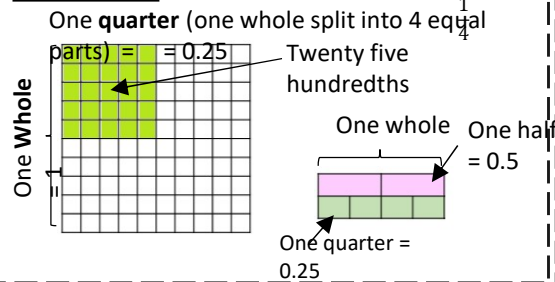
### Percentages on a hundred grid



100% = a whole = 100 hundredths  
 7 hundredths  
 7 out of 100  
 7% hundredths

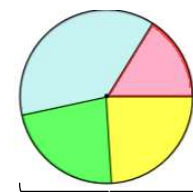
6 tenths and 3 hundredths  
 63 hundredths  
 63%

### Quarters



One quarter (one whole split into 4 equal parts) =  $\frac{1}{4} = 0.25$   
 Twenty five hundredths  
 One whole  
 One half = 0.5  
 One quarter = 0.25

### Simple pie charts

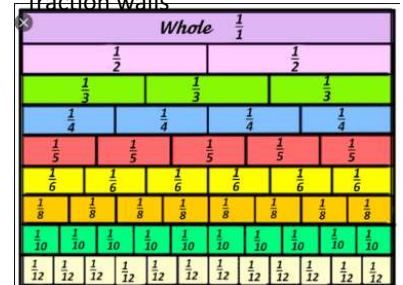


A pie chart has 360° so all FDP calculations are out of 360

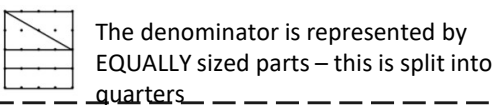
- Split into 10 parts = 10% = 36°
- Split into 2 parts = 50% = 180°
- Split into 5 parts = 20% = 72°

### Equivalent fractions

Represent equivalence with fraction walls

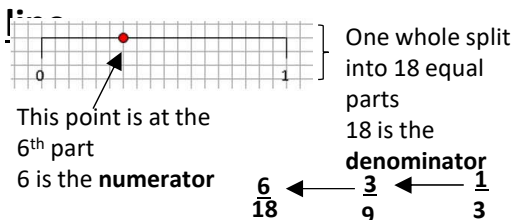


### Fractions – on a diagram



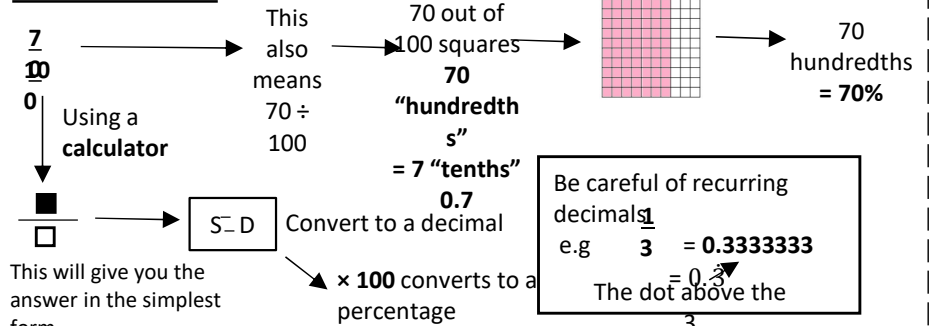
The denominator is represented by EQUALLY sized parts – this is split into quarters

### Fractions – on a number



One whole split into 18 equal parts  
 This point is at the 6th part  
 6 is the numerator  
 18 is the denominator  
 $\frac{6}{18} = \frac{1}{3}$

### Convert FDP



$\frac{7}{10}$  → This also means 70 out of 100 squares → 70 hundredths = 70%  
 Using a calculator:  $70 \div 100 = 0.7$   
 Convert to a decimal:  $\frac{70}{100} = 0.7$   
 × 100 converts to a percentage  
 Be careful of recurring decimals! e.g.  $\frac{1}{3} = 0.333333$   
 The dot above the 3

<p><b>Question 1</b></p> <p>Express <math>\frac{19}{25}</math> as a percentage</p>	<p><b>Question 2</b></p> <p>Express <math>\frac{9}{50}</math> as a decimal</p>	<p><b>Question 3</b></p> <p>Complete the equivalent fraction:</p> $\frac{2}{3} = \frac{\quad}{12}$	<p><b>Question 4</b></p> <p>Complete the equivalent fraction:</p> $\frac{6}{7} + \frac{\quad}{28}$
<p><b>Question 5</b></p> <p>Express <math>\frac{13}{20}</math> as a decimal</p>	<p><b>Question 6</b></p> <p>Express <math>\frac{22}{25}</math> as a percentage</p>	<p><b>Question 7</b></p> <p>Complete the equivalent fraction:</p> $\frac{3}{8} = \frac{\quad}{56}$	<p><b>Question 8</b></p> <p>Complete the equivalent fraction:</p> $\frac{8}{9} = \frac{\quad}{81}$
<p><b>Question 9</b></p> <p>Express 60 as a fraction</p>	<p><b>Question 10</b></p> <p>Express 74 as a fraction</p>	<p><b>Question 11</b></p> <p>Complete the equivalent fraction:</p> $\frac{3}{4} = \frac{\quad}{28}$	<p><b>Question 12</b></p> <p>Complete the equivalent fraction:</p> $\frac{4}{7} = \frac{\quad}{21}$
<p><b>Question 13</b></p> <p>Express 0.1 as a fraction</p>	<p><b>Question 14</b></p> <p>Express <math>\frac{47}{50}</math> as a decimal</p>	<p><b>Question 15</b></p> <p>Complete the equivalent fraction:</p> $\frac{2}{3} = \frac{\quad}{24}$	<p><b>Question 16</b></p> <p>Complete the equivalent fraction:</p> $\frac{5}{8} = \frac{\quad}{24}$
<p><b>Question 17</b></p> <p>Express <math>\frac{7}{10}</math> as a decimal</p>	<p><b>Question 18</b></p> <p>Express <math>\frac{1}{25}</math> as a percentage</p>	<p><b>Question 19</b></p> <p>Complete the equivalent fraction:</p> $\frac{5}{9} = \frac{\quad}{27}$	<p><b>Question 20</b></p> <p>Complete the equivalent fraction:</p> $\frac{4}{7} = \frac{\quad}{84}$

Answers: 1. 76%, 2. 0.18, 3. 8, 4. 24, 5. 0.65, 6. 88%, 7. 21, 8. 72, 9.  $\frac{3}{5}$ , 10.  $\frac{37}{50}$ , 11. 21, 12. 12, 13.  $\frac{1}{10}$ , 14. 0.94, 15. 16, 16. 15, 17. 0.7, 18. 4%, 19. 15, 20. 48

## Ordering integers and decimals

### What do I need to be able to do?

By the end of this unit you should be able to:

- Understand place value and the number system including decimals
- Understand and use place value for decimals, integers and measures of any size
- Order number and use a number line for positive and negative integers, fractions and decimals;
- use the symbols =, ≠, ≤, ≥
- Work with terminating decimals and their corresponding fractions
- Round numbers to an appropriate accuracy
- Describe, interpret and compare data distributions using the median and range

### Keywords

- Approximate:** To estimate a number, amount or total often using rounding of numbers to make them easier to calculate with
- Integer:** a whole number that is positive or negative
- Interval:** between two points or values
- Median:** A measure of central tendency (middle, average) found by putting all the data values in order and finding the middle value of the list.
- Negative:** Any number less than zero; written with a minus sign.
- Place holder:** We use 0 as a place holder to show that there are none of a particular place in a number
- Place value:** The value of a digit depending on its place in a number. In our decimal number system, each place is 10 times bigger than the place to its right
- Range:** The difference between the largest and smallest numbers in a set
- Significant figure:** A digit that gives meaning to a number. The most significant digit (figure) in an integer is the number on the left. The most significant digit in a decimal fraction is the first non-zero number after the decimal point.

### Integer Place Value

Billions			Millions			Thousands			Ones		
H	T	O	H	T	O	H	T	O	H	T	O
		3	1	4	8	0	3	3	0	2	9

Placeholder

Three billion, one hundred and forty eight million, thirty three thousand and twenty nine

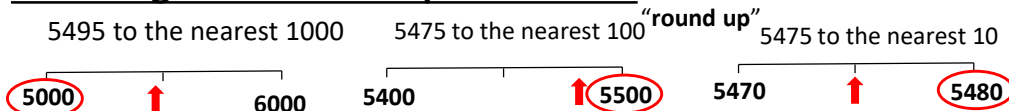
1 billion 1, 000, 000, 000

1 million 1, 000, 000

### Intervals on a number line



### Rounding to the nearest power of ten



### Compare integers using <, >, =, ≠

- < less than
- > greater than
- = equal to
- ≠ not equal to
- Two and a half million (2 500 000)
- Three billion (3 000 000 000)
- One thousand and eighty (68 000)

### Range Spread of the values

Difference between the biggest and smallest

3 9 8 12

Range: Biggest value – Smallest value

12 – 3 = 9

Range = 9

### Median The middle value

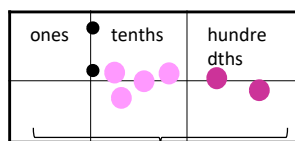
**Example 1** Median: put the in order 3 4 8 9 12  
4 3 9 8 12 find the middle number 3 4 8 9 12

**Example 2** Median: put the in order 150 154 148 137 148 150 154 158 160  
150 154 148 137 160 158  
There are 2 middle numbers 150 154 158 160  
Find the midpoint 152

### Decimals

We say "nought point five two"

Five tenths and two hundredths



$$0 \text{ ones, } 5 \text{ tenths and } 2 \text{ hundredths}$$

$$= 0 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.01 + 0.01$$

$$= 0 + 0.5 + 0.02$$

$$= 0.52$$

### Comparing decimals

Which is the largest of 0.3 and 0.23?

$$0.3 > 0.23$$

"There are more counters in the furthest column to the left"

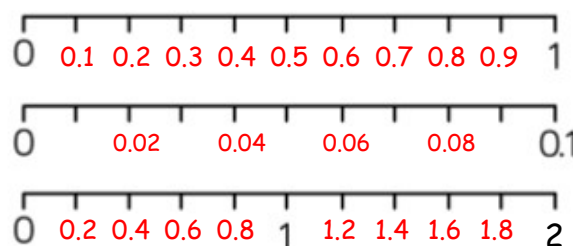
0.3  
0  
0.2  
3

Comparing the values both with the same number of decimal places is another way to compare the number of tenths and hundredths

One s	Tenths	hundredths
	0.1	0.1
	0.1	
One s	Tenths	hundredths
	0.1	0.01
	0.1	0.01

### Decimal intervals on a number line

One whole split into 10 parts makes tenths = 0.1  
One tenth split into 10 parts makes hundredths = 0.01



### Round to 1 significant figure

- 370 to 1 significant figure is 400
- 37 to 1 significant figure is 40
- 3.7 to 1 significant figure is 4
- 0.37 to 1 significant figure is 0.4
- 0.00000037 to 1 significant figure is 0.0000004

Round to the first non zero number

<p><b>Question 1</b></p> <p>Write in ascending order: 0.06,0.5,0.65</p>	<p><b>Question 2</b></p> <p>Write in ascending order: 0.7,0.08,0.82</p>	<p><b>Question 3</b></p> <p>Round 50 correct to 1 significant figure</p>	<p><b>Question 4</b></p> <p>Round 0.06574 correct to 1 significant figure</p>
<p><b>Question 5</b></p> <p>Write in figures: Four hundred million and twenty two thousand</p>	<p><b>Question 6</b></p> <p>Write in figures: Three hundred million one hundred and two thousand</p>	<p><b>Question 7</b></p> <p>Find the median: 6,12,3,11,6</p>	<p><b>Question 8</b></p> <p>Find the median: 7,10,7,12,7,12</p>
<p><b>Question 9</b></p> <p>Write in ascending order: 3.26, 0.437, 3.07</p>	<p><b>Question 10</b></p> <p>Write in ascending order: 0.17, 0.2, 0.02</p>	<p><b>Question 11</b></p> <p>Round 0.0032227 correct to 1 significant figure</p>	<p><b>Question 12</b></p> <p>Round 0.0218 correct to 1 significant figure</p>
<p><b>Question 13</b></p> <p>Write in figures: sixty three million six hundred</p>	<p><b>Question 14</b></p> <p>Write in figures: seven hundred and nine million and one</p>	<p><b>Question 15</b></p> <p>Write in ascending order: 0.8, 0.08, 0.96</p>	<p><b>Question 16</b></p> <p>Find the range: 12,3,5,6,12,15,3</p>
<p><b>Question 17</b></p> <p>Find the range: 6,15,10,15,6,6,6</p>	<p><b>Question 18</b></p> <p>Round 6762 correct to 1 significant figure</p>	<p><b>Question 19</b></p> <p>Round 5059 correct to 1 significant figure</p>	<p><b>Question 20</b></p> <p>Find the median: 13,15,7,10,14</p>

Answers: 1. 0.06,0.5,0.65, 2. 0.08,0.7,0.82, 3. 50, 4. 0.07, 5. 400022000, 6. 300102000, 7. 6, 8. 8.5, 9. 0.437,3.07,3.26, 10. 0.02,0.17,0.2, 11. 0.003, 12. 00.02, 13. 63000600, 14. 709000001, 15.0.08,0.8,0.96, 16. 12, 17. 9, 18. 7000, 19. 5000, 20. 13



# Year 7 – application of number

## Solving problems with addition and subtraction



### What do I need to be able to do?

By the end of this unit you should be able to:

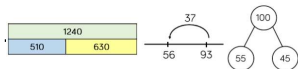
- Understand properties of addition/ subtraction
- Use mental strategies for addition/subtraction
- Use formal methods of addition/Subtraction for integers
- Use formal methods of addition/Subtraction for decimals
- Solve problems in context of perimeter
- Solve problems with finance, tables and timetables
- Solve problems with frequency trees
- Solve problems with bar charts and line charts

### Keywords

- Commutative:** changing the order of the operations does not change the result
- Associative:** when you add or multiply you can do so regardless of how the numbers are grouped
- Inverse:** the operation that undoes what was done by the previous operation. (The opposite operation)
- Placeholder:** a number that occupies a position to give value
- Perimeter:** the distance/ length around a 2D object
- Polygon:** a 2D shape made with straight lines
- Balance:** in financial questions – the amount of money in a bank account
- Credit:** money that goes into a bank account
- Debit:** money that leaves a bank account

### Addition/ Subtraction with

Addition is commutative same



$$6 + 3 = 3 + 6$$

The order of addition does not change the result

Modelling methods for addition/ subtraction

- Bar models
- Number lines
- Part/ Whole diagrams

Subtraction the order has to stay the

$$360 - 147 = 360 - 100 - 40 - 7$$

Formal written methods

	H	T	O
	1	8	7
+	5	4	2

	H	T	O
	4	2	7
-	2	4	9

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction
- Show your relationships by writing fact families

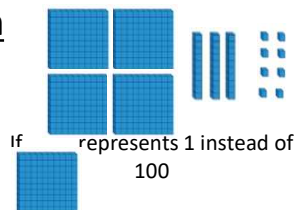
Remember the place value of each column.  
You may need to move 10 ones to the ones column to be able to subtract

### Addition/ Subtraction with decimals

4	.	3	8
7	.	9	0
			+

0 can be used to fill empty places with value

The decimal place acts as the placeholder and aligns the other values



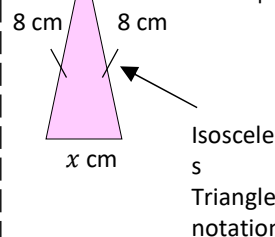
If represents 1 instead of 100

$$5.43 + \frac{8}{10}$$

Revisit Fraction – Decimal equivalence  
 $5.43 + 0.8$

### Solve problems with perimeter

Perimeter is the length around the outside of a polygon



The triangle has a perimeter of 25cm. Find the length of x

$$8\text{cm} + 8\text{cm} + x\text{cm} = 25\text{cm}$$

$$16\text{cm} + x\text{cm} = 25\text{cm}$$

$$x\text{cm} = 9\text{cm}$$

Isosceles Triangle notation

### Solve problems with

$$\text{Profit} = \text{Income} - \text{Costs}$$

Credit – Money coming into an account

Debit – Money leaving an account

Money uses a two decimal place system.  
14.2 on a calculator represents £14.20

Check the units of currency – work in the same unit

### Tables and timetables

Distance tables

London

211	Cardiff		
556	493	Glasgow	
518	392	177	Belfast

This shows the distance between Glasgow and London. It is where their row and column intersects

Bus/ Train timetables

Harton	1005	1045	1130
Bridge	1024	1106	1147
Aville	1051	1133	1205
Ware	1117	1202	1233

Each column represents a journey, each row represents the time the 'bus' arrives at that location  
**TIME CALCUALTIONS** – use a number line

Two-way tables

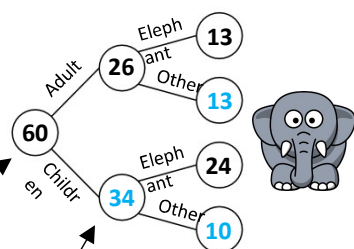
	H	T
H	HH	HT
T	TH	TT

Where rows and columns intersect is the outcome of that action.

### Frequency trees

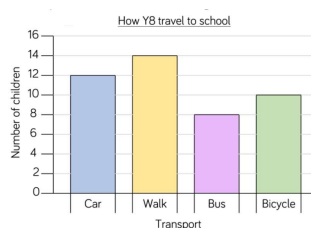
60 people visited the zoo one Saturday morning.  
26 of them were adults. 13 of the adult's favourite animal was an elephant. 24 of the children's favourite animal was an elephant. The overall total "60 people"

A frequency tree is made up from part-whole models. One piece of information leads to another



Probabilities or statements can be taken from the completed trees e.g. 34 children visited the zoo

### Bar and line charts



Use addition/ subtraction methods to extract information from bar charts.

e.g. Difference between the number of students who walked and took the bus.  
Walk frequency – bus frequency

When describing changes or making predictions.

- Extract information from your data source
- Make comparisons of difference or sum of values.
- Put into the context of the scenario

<p><b>Question 1</b></p> <p>Complete <math>11018 + \underline{\hspace{2cm}} = 13873</math></p>	<p><b>Question 2</b></p> <p>Complete <math>\underline{\hspace{2cm}} - 2121 = 18021</math></p>	<p><b>Question 3</b></p> <p>Calculate <math>47.1 - 2</math></p>	<p><b>Question 4</b></p> <p>Calculate <math>7.19 - 0.23</math></p>
<p><b>Question 5</b></p> <p>Complete <math>8765 - \underline{\hspace{2cm}} = 8490</math></p>	<p><b>Question 6</b></p> <p>Complete <math>\underline{\hspace{2cm}} - 19235 = 13146</math></p>	<p><b>Question 7</b></p> <p>Calculate <math>3.36 + 88.6</math></p>	<p><b>Question 8</b></p> <p>Calculate <math>0.466 + 16</math></p>
<p><b>Question 9</b></p> <p>Complete <math>\underline{\hspace{2cm}} 19905 = 12063</math></p>	<p><b>Question 10</b></p> <p>Complete <math>11969 + \underline{\hspace{2cm}} = 207711</math></p>	<p><b>Question 11</b></p> <p>Calculate <math>6.18 + 0.471</math></p>	<p><b>Question 12</b></p> <p>Calculate <math>63.5 + 0.265</math></p>
<p><b>Question 13</b></p> <p>Complete <math>12445 - \underline{\hspace{2cm}} = 12311</math></p>	<p><b>Question 14</b></p> <p>Complete <math>2540 + \underline{\hspace{2cm}} = 16107</math></p>	<p><b>Question 15</b></p> <p>Calculate <math>60.1 - 0.835</math></p>	<p><b>Question 16</b></p> <p>Calculate <math>49.1 - 46.7</math></p>
<p><b>Question 17</b></p> <p>Complete <math>4404 + \underline{\hspace{2cm}} = 8665</math></p>	<p><b>Question 18</b></p> <p>Complete <math>2701 + \underline{\hspace{2cm}} = 12547</math></p>	<p><b>Question 19</b></p> <p>Calculate <math>26.8 - 9.35</math></p>	<p><b>Question 20</b></p> <p>Calculate <math>4.8 - 0.586</math></p>

Answers: 1. 2855, 2. 20142, 3. 45.1, 4. 6.96, 5. 275, 6. 32381, 7. 91.96, 8. 16.466, 9. 31968, 10. 8802, 11. 6.651, 12. 63.765, 13. 134, 14. 13567, 15. 59.265, 16. 2.4, 17. 4261, 18. 9846, 19. 17.45, 20. 4.214

# Year 7 – application of number

## Fractions and percentages of amounts



### What do I need to be able to do?

By the end of this unit you should be able to:

- Find a fraction of a given amount
- Use a given fraction to find the whole or other fractions
- Find the percentage of an amount using mental methods
- Find the percentage of a given amount using a calculator

### Keywords

**Fraction:** how many parts of a whole we have

**Equivalent:** of equal value

**Whole:** a number with no fractional or decimal part.

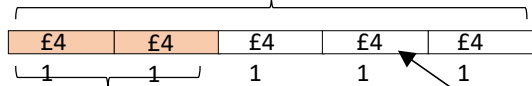
**Percentage:** parts per 100 (uses the % symbol)

**Place Value:** the value of a digit depending on its place in a number. In our decimal number system, each place is 10 times bigger than the place to its right

**Convert:** change into an equivalent representation, often fraction to decimal to a percentage cycle.

### Fraction of a given amount

Find  $\frac{2}{5}$  of £205



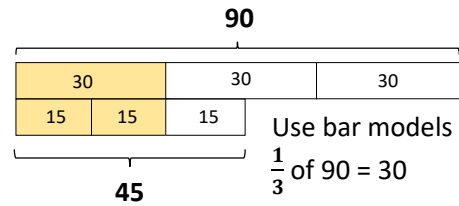
2 out of the 5 equal parts

$$2 \times £41 = \underline{£82}$$

The bar represents the whole amount

$$£205 \div 5 =$$

Each part of the bar model represents £41.



Use bar models for comparisons

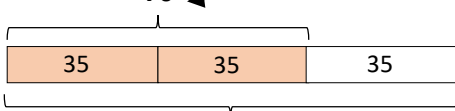
$$\frac{1}{3} \text{ of } 90 = 30$$

$$\frac{2}{3} \text{ of } 45 = 30$$

$$\therefore \frac{1}{3} \text{ of } 90 = \frac{2}{3} \text{ of } 45$$

### Use a fraction of amount

$\frac{2}{3}$  of a value is 70. What is the whole number?



$$35 \times 3 = 105$$

The whole number is 105

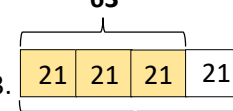
$$70 \div 2 = 35$$

Each part of the bar model represents 35.

The wording of the question is important to setting up

the bar model

$\frac{3}{4}$  of a number is 63.



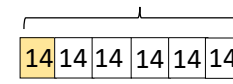
Find the whole

What is  $\frac{1}{6}$  of the number?

$$= 1$$

$$4$$

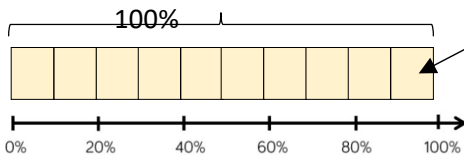
84



Use the whole to find a given part

### Find the percentage of an amount (Mental methods)

The whole represents



$10\% = \frac{1}{10}$  of the whole

$10\% = \frac{1}{10}$  of the whole

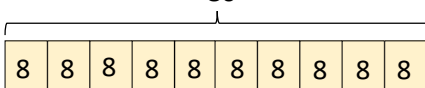
$50\% = \frac{5}{10} = \frac{1}{2}$  of the whole

$20\% = \frac{2}{10} = \frac{1}{5}$  of the whole

$5\% = \frac{1}{20}$  of the whole

Find 65% of 80

80



For bigger percentages it is sometimes easier to take away from 100%

Method 1:  
 $65\% = 10\% \times 6 + 5\%$   
 $= (8 \times 6) + 4$   
 $= 52$

Method 2:  
 $65\% = 50\% + 10\% + 5\%$   
 $= 40 + 8 + 4$   
 $= 52$

### Find the percentage of an amount (Calculator methods)



Using a multiplier

Find 65% of 80

Fraction, decimal, percentage conversion

$$65\% = \frac{65}{100} = 0.65$$

The multiplier

$$0.65 \times 80 = 52$$

Using the percent button

Find 65% of 80

This brings up the % button on screen

You will see 65%

Type 65

Pres: **SHIFT** **(%)**

Pres: **x** 80 and then press =

"of" can represent 'x' in calculator methods

You can also use the calculator to support non calculator methods and find 1% or 10% then add percentages together

Question 1 Calculate $\frac{3}{14}$ of £560	Question 2 Calculate $\frac{3}{7}$ of £511	Question 3 Calculate $\frac{5}{13}$ of £377	Question 4 Calculate $\frac{4}{11}$ of £297
Question 5 Calculate $\frac{3}{4}$ of £64	Question 6 Calculate $\frac{1}{14}$ of £854	Question 7 Calculate $\frac{7}{9}$ of £135	Question 8 Calculate $\frac{7}{13}$ of £481
Question 9 Calculate $\frac{2}{3}$ of £138	Question 10 Calculate $\frac{11}{13}$ of £377		

Answers: 1. £120, 2. £219, 3. £145, 4. £108, 5. £48, 6. £61, 7. £105, 8. £259, 9. £92, 10. £319

# Year 7 – application of number

## Solving problems with multiplication and division



### What do I need to be able to do?

By the end of this unit you should be able to:

- Understand and use factors
- Understand and use multiples
- Multiply/ Divide integers and decimals by powers of 10
- Use formal methods to multiply
- Use formal methods to divide
- Understand and use order of operations
- Solve area problems
- Solve problems using the mean

### Keywords

- Array:** an arrangement of items to represent concepts in rows or columns  
**Multiples:** found by multiplying any number by positive integers  
**Factor:** integers that multiply together to get another number.  
**Mili:** prefix meaning one thousandth  
**Centi:** prefix meaning one hundredth.  
**Kilo:** prefix meaning multiply by 1000  
**Quotient:** the result of a division  
**Dividend:** the number being divided  
**Divisor:** the number we divide by.

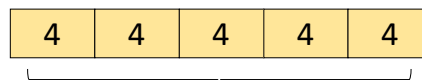
### Factors

●●●● Arrays can help represent factors  
 $5 \times 2$  or  $2 \times 5$        $10 \times 1$  or  $1 \times 10$   
**Factors of 10**  
 1, 2, 5, 10  
 The number itself is always a factor

**Square numbers** have an **ODD number** of factors

**Factors of 4**      **Factors of 36**  
 1, 2, 4      1, 2, 3, 4, 6, 9, 12, 18, 36  
 Be strategic - Lay factors out in pairs can help you not to miss any

### Multiples



Bar models can represent by something is a multiple. E.g. 20 is a multiple of 4

### Lowest Common Multiples

**9**    9, 18, 27, **36**, 45, 54  
**12**    12, 24, **36**, 48, 60

The first time the multiples match  
**LCM = 36**



### Multiply/ Divide by powers of 10



$3 \times 100 = 300$

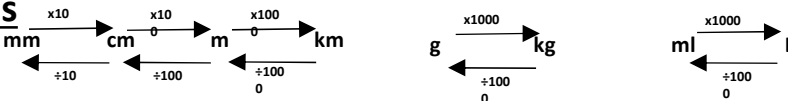


$0.03 \times 100 = 3$

Repeated multiplication and division by powers of 10 is commutative  
 $\div 10$  then  $\div 10 \rightarrow \div 100$

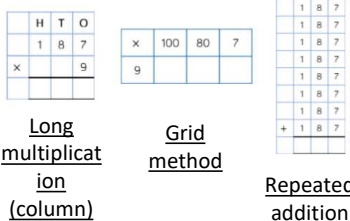
### Metric conversions

Useful Conversions



### Multiplication methods

Less effective method especially for bigger multiplication



### Multiplication with decimals

Perform multiplications as integers

e.g.  $0.2 \times 0.3 \rightarrow 2 \times 3$

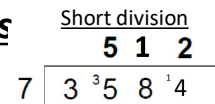
Make **adjustments** to your answer to match the question:  
 $0.2 \times 10 = 2$

$0.3 \times 10 = 3$

Therefore  $6 \div 100 = 0.6$

### Division methods

$3584 \div 7 = 512$



### Complex division

$\div 24 = \div 6 \div 4$

Break up the divisor using factors

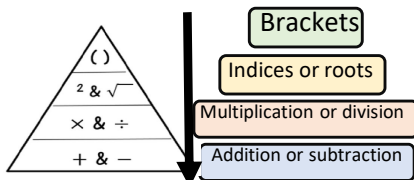
### Division with decimals

The placeholder in division methods is essential – the decimal lines up on the dividend and the quotient

$2.4 \div 0.02 \rightarrow 24 \div 0.2 \rightarrow 240 \div 2$

All give the same solution as represent the same proportion. Multiply the values in proportion until the divisor becomes an integer

### Order of operations



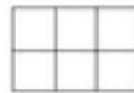
If you have multiple operations from the same tier work from left to right

e.g.  $10 - 3 + 7 + 5$

$6 \times 4 + 8 \times 2$   
 $24 + 16 = 40$

### Area problems

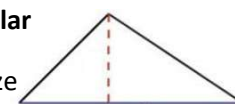
Rectangle  
**Base x Perpendicular height**



Parallelogram/ Rhombus  
**Base x Perpendicular height**



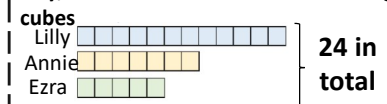
Triangle  
 $\frac{1}{2} \times \text{Base} \times \text{Perpendicular height}$   
 A triangle is half the size of the rectangle it would fit in



### Mean problems

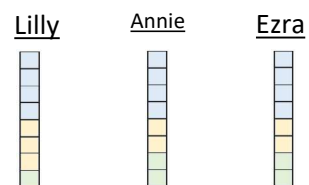
Mean – a measure of average  
 It gives an idea of the central value

Lilly, Annie and Ezra have the following



**24 in total**

Finding the mean amount is the average amount each person would have if shared out equally



The mean number of blocks would be 8 each

<p><b>Questions 1</b></p> <p>What is the highest common factor of 77 and 147?</p>	<p><b>Questions 2</b></p> <p>What is the highest common factor of 55 and 50</p>	<p><b>Questions 3</b></p> <p>Find the lowest multiple of 18 and 15</p>	<p><b>Questions 4</b></p> <p>Find the lowest common multiple of 30 and 6</p>
<p><b>Questions 5</b></p> <p>Work out <math>93.9 \times 10000 =</math></p>	<p><b>Questions 6</b></p> <p>Work out <math>9710 \times 1000 =</math></p>	<p><b>Questions 7</b></p> <p>Work out <math>0.174 \div 100 =</math></p>	<p><b>Questions 8</b></p> <p>Complete: <math>4\text{km} = \underline{\hspace{2cm}}\text{m}</math></p>
<p><b>Questions 9</b></p> <p>Complete: <math>5000\text{m} = \underline{\hspace{2cm}}\text{km}</math></p>	<p><b>Questions 10</b></p> <p>Work out <math>632 \div 10000 =</math></p>	<p><b>Questions 11</b></p> <p>Work out <math>1480 \times 0.1 =</math></p>	<p><b>Questions 12</b></p> <p>What is the higher common factor of 120 and 72?</p>
<p><b>Questions 13</b></p> <p>What is the highest common factor of 134 and 67?</p>	<p><b>Questions 14</b></p> <p>Find the lowest common multiple of 24 and 8</p>	<p><b>Questions 15</b></p> <p>Find the lowest common multiple of 22 and 11</p>	<p><b>Questions 16</b></p> <p>Work out <math>0.029 \times 1000 =</math></p>
<p><b>Questions 17</b></p> <p>Work out <math>0.7886 \times 10 =</math></p>	<p><b>Questions 18</b></p> <p>Work out <math>362 \div 1000 =</math></p>	<p><b>Questions 19</b></p> <p>Complete: <math>9\text{m} = \underline{\hspace{2cm}}\text{cm}</math></p>	<p><b>Questions 20</b></p> <p>Complete: <math>5000\text{ml} = \underline{\hspace{2cm}}\text{litres}</math></p>

Answers: 1. 7, 2. 5, 3. 90, 4. 30, 5. 939000, 6. 9710000, 7. 0.00174, 8. 4000, 9. 5, 10. 0.0632, 11. 148, 12. 24, 13. 67, 14. 24, 15. 22, 16. 29, 17. 7.86, 18. 0.362, 19. 900, 20. 5

# Year 7 – directed number

## Operations with equations and directed numbers

### What do I need to be able to do?

By the end of this unit you should be able to:

- Perform calculations that cross zero
- Add/ Subtract directed numbers
- Multiply/ Divide directed numbers
- Evaluate algebraic expressions
- Solve two-step equations
- Use order of operations with directed number

### Keywords

- Subtract:** taking away one number from another.
- Negative:** a value less than zero.
- Commutative:** changing the order of the operations does not change the result
- Product:** multiply terms
- Inverse:** the opposite function
- Square root:** a square root of a number is a number when multiplied by itself gives the value (symbol  $\sqrt{\quad}$ )
- Square:** a term multiplied by itself.
- Expression:** a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

### Perform calculations that cross zero

Number lines are useful to help you visualise the calculation crossing 0

$4 - 6 = -2$

Use the number line to guide subtraction of 6

Start at 4

Find the difference between 6 and -4

From 6 to 0: 6

From 0 to -4: 4

10 beads between them

$-5 + 5 = 0$

Rearrangements of the same equation

$5 - 5 = 0$

### Add directed number

$2 + -4 = -2$

Representation: Red = -1, Yellow = 1

Zero pair (-1 + 1 = 0)

Two "-1" left = -2

$8 + -3 = 5$

Partitioning:  $8 + -3 = 5$  becomes  $5 + 3 + -3 = 5$

Generalisation:  $+ - = -$

Partition the value to create a zero pair calculation

### Subtract directed numbers

Representation for calculation: Red = -1, Yellow = 1

"Subtract" - means take away or remove

$2 - -1 = 3$

Take away 1

Start with the representation of 2

$2 - -3 = 5$

Generalisation:  $- - = +$

### Multiply/ Divide directed

Two representations of the same calculation:  $-3 \times -3 = 9$

$2 \times -3 = -6$

$-2 \times -3 = 6$

Negative, Negative calculation

This is the negative of 2

$x - 3$

Divisions are the inverse operations

The act of making counters into their negative is turning them over

### Evaluate algebraic expressions

$a = 5$ ,  $b = -4$

$a^2 = 5^2 = 25$

$b^2 = (-4)^2 = 16$

With negative numbers the brackets are important so that it performs  $-4 \times -4$ .

Brackets around negative substitutions helps remove calculation errors

$2a - b = 2 \times 5 - (-4) = 10 + 4 = 14$

$3b - 2a = 3(-4) - 2(5) = -12 - 10 = -22$

### Two-step equations

Bar Model:  $4x + 2 = 10$

$10 - 4x = 2$

Representing the same question (use fact families)

Function machine:  $x \rightarrow x4 \rightarrow +2 \rightarrow 10$

Inverse operations to find x

### Use order of operations

Order of operations: Brackets, Indices or roots, Multiplication or division, Addition or subtraction

Remember square roots have a positive and negative value

Brackets around negative substitutions helps remove calculation errors

x	-5	-2	-1	0	1	2	3
-3	9	6	3	0	-3	-6	-9
-2	6	4	2	0	-2	-4	-6
-1	3	2	1	0	-1	-2	-3
0	0	0	0	0	0	0	0
1	-3	-2	-1	0	1	2	3
2	-6	-4	-2	0	2	4	6
3	-9	-6	-3	0	3	6	9

<p>Question 1</p> <p>Work out <math>-8-3</math></p>	<p>Question 2</p> <p>Work out <math>-1- - 1</math></p>	<p>Question 3</p> <p>Work out <math>42 \div -6 =</math></p>	<p>Question 4</p> <p>Work out <math>24 \div -6 =</math></p>
<p>Question 5</p> <p>Solve <math>5x - 10 = 5</math></p>	<p>Question 6</p> <p>Solve <math>3x - 10 = 20</math></p>	<p>Question 7</p> <p>Find the missing number  <math>? - -5 = 8</math></p>	<p>Question 8</p> <p>Find the missing number  <math>? - 2 = -8</math></p>
<p>Question 9</p> <p>Work out <math>-4 + 2</math></p>	<p>Question 10</p> <p>Work out <math>2 - -3</math></p>	<p>Question 11</p> <p>Work out <math>-3 \times -9 =</math></p>	<p>Question 12</p> <p>Work out <math>44 \div -11 =</math></p>
<p>Question 13</p> <p>Solve <math>11x + 10 = 98</math></p>	<p>Question 14</p> <p>Solve <math>2x - 8 = 0</math></p>	<p>Question 15</p> <p>Find the missing number  <math>? + 1 = -5</math></p>	<p>Question 16</p> <p>Find the missing number <math>1 - ? = 7</math></p>
<p>Question 17</p> <p>Work out <math>- 1 + - 6</math></p>	<p>Question 18</p> <p>Work out <math>4 - -3</math></p>	<p>Question 19</p> <p>Find the value of <math>x + 8</math>, when <math>x = -6</math></p>	<p>Question 20</p> <p>Find the value of <math>7x - 11</math>, when <math>x = -1</math></p>

Answers: 1. -11 2. 0 3. -7 4. -4 5. 3 6. 10 7. 3 8. -6 9. -2 10. 5 11. 27 12. -4 13. 8 14. 4 15. -6 16. -6 17. -7 18. 7 19. 2 20. -18



# Year 7 – Fractional Thinking

## Addition and subtraction of fractions

### What do I need to be able to do?

By the end of this unit you should be able to:

- Convert between mixed numbers and fractions
- Add/Subtract unit fractions (same denominator)
- Add/Subtract fractions (same denominator)
- Add/Subtract fractions from integers
- Use equivalent fractions
- Add/Subtract any fractions
- Add/Subtract improper fractions and mixed numbers
- Use fractions in algebraic contexts

### Keywords

**Numerator** : the number above the line on a fraction. The top number. Represents how many parts are taken

**Denominator**: the number below the line on a fraction. The number represent the total number of parts

**Equivalent**: of equal value

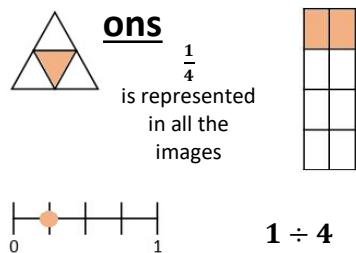
**Mixed numbers**: a number with an integer and a proper fraction

**Improper fractions**: a fraction with a bigger numerator than denominator

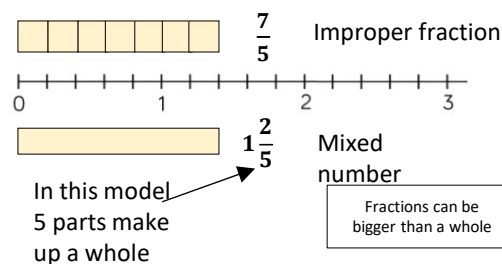
**Substitute**: replace a variable with a numerical value

**Place value**: the value of a digit depending on its place in a number. In our decimal number system, each place is 10 times bigger than the place to its right

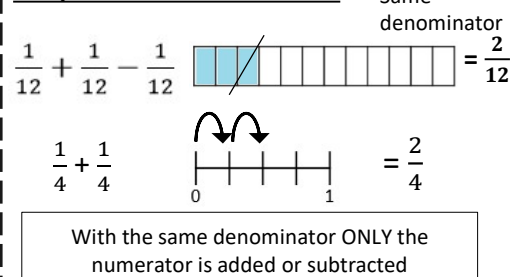
### Representing



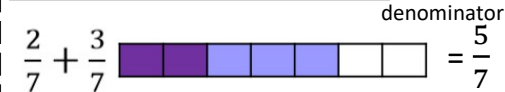
### Mixed numbers and fractions



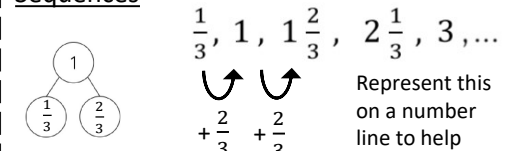
### Add/Subtract unit fractions



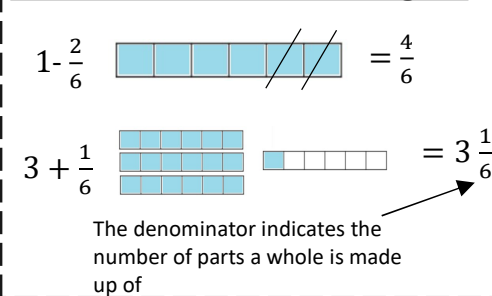
### Add/Subtract fractions



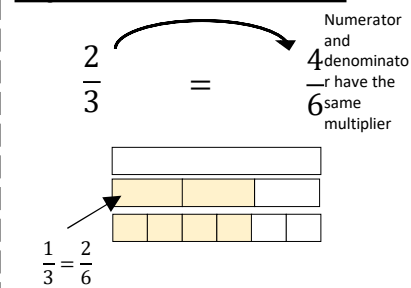
### Sequences



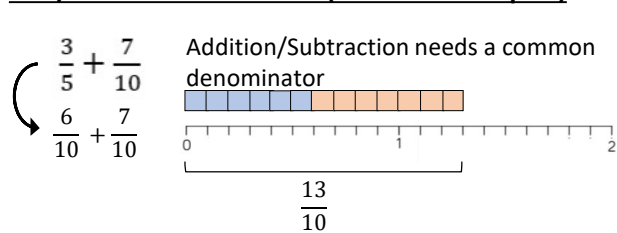
### Add/Subtract from integers



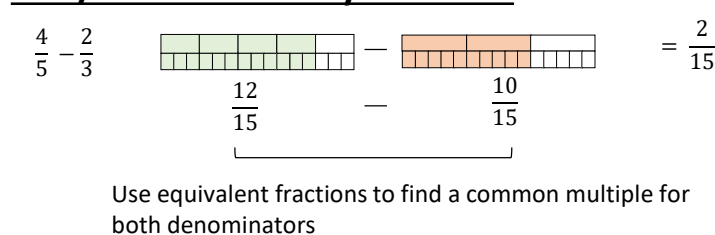
### Equivalent fractions



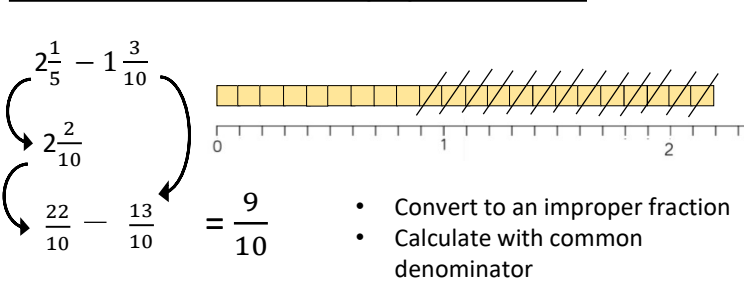
### Add/Subtraction fractions (common multiples)



### Add/Subtraction any fractions



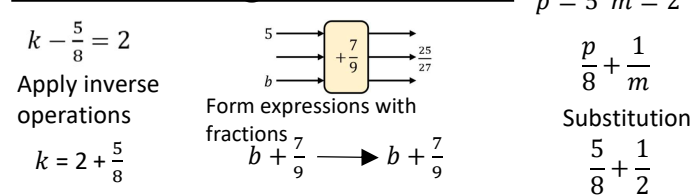
### Add/Subtraction fractions (improper and mixed)



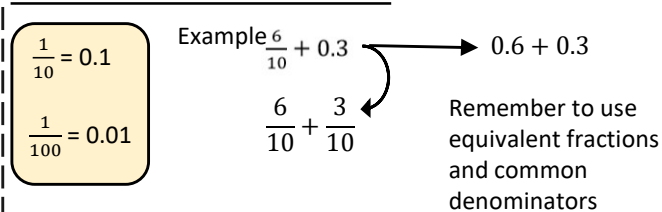
### Partitioning method

$$2\frac{1}{5} - 1\frac{3}{10} = 2\frac{2}{10} - 1\frac{3}{10} = 2\frac{2}{10} - 1 - \frac{3}{10} = 1\frac{2}{10} - \frac{3}{10} = \frac{9}{10}$$

### Fractions in algebraic contexts



### Fractions and decimals



Question 1 Express $\frac{23}{50}$ as a decimal	Question 2 Express $\frac{43}{3}$ as a mixed number	Question 3 Express $\frac{24}{7}$ as mixed number	Question 4 Work out $\frac{4}{5} - \frac{3}{8}$
Question 5 Work out $\frac{3}{4} - \frac{3}{5}$	Question 6 Express $\frac{1}{5}$ as a percentage	Question 7 Express $\frac{1}{10}$ as a percentage	Question 8 Express $\frac{30}{13}$ as a mixed number
Question 9 Express $5\frac{1}{6}$ as an improper fraction	Question 10 Work out $\frac{3}{5} + \frac{1}{3}$	Question 11 Work out $\frac{1}{3} - \frac{2}{7}$	Question 12 Express $\frac{2}{5}$ as a percentage
Question 13 Express $\frac{27}{50}$ as a decimal	Question 14 Express $\frac{41}{9}$ as a mixed number	Question 15 Express $\frac{35}{6}$ as a mixed number	Question 16 Work out $\frac{3}{7} + \frac{1}{4}$
Question 17 Work out $\frac{3}{4} + \frac{5}{7}$	Question 18 Express $\frac{1}{25}$ as a percentage	Question 19 Express $\frac{3}{4}$ as a percentage	Question 20 Express $1\frac{11}{13}$ as an improper fraction

Answers: 1. 0.46, 2.  $14\frac{1}{3}$ , 3.  $3\frac{3}{7}$ , 4.  $\frac{17}{40}$ , 5.  $\frac{3}{20}$ , 6. 20%, 7. 10%, 8.  $2\frac{4}{13}$ , 9.  $\frac{31}{6}$ , 10.  $\frac{14}{15}$ , 11.  $\frac{1}{21}$ , 12. 40%, 13. 0.54, 14.  $4\frac{5}{9}$ , 15.  $5\frac{5}{6}$ , 16.  $\frac{19}{28}$ , 17.  $1\frac{13}{28}$ , 18. 4%, 19. 75%, 20.  $\frac{24}{13}$

# Year 7 – lines and angles

## Constructing, measuring and using geometric notation

### What do I need to be able to do?

By the end of this unit you should be able to:

- Use letter and labelling conventions
- Draw and measure line segments and angles
- Identify parallel and perpendicular lines
- Recognise types of triangle
- Recognise types of quadrilateral
- Identify polygons
- Construct triangles (SAS, SSS, ASA)
- Draw Pie charts

### Keywords

- Polygon:** A 2D shape made with straight lines  
**Scalene triangle:** a triangle with all different sides and angles  
**Isosceles triangle:** a triangle with two angles the same size and two angles the same size  
**Right-angled triangle:** a triangle with a right angle  
**Frequency:** the number of times a data value occurs  
**Sector:** part of a circle made by two radii touching the centre  
**Rotation:** turn in a given direction  
**Protractor:** equipment used to measure angles  
**Compass:** equipment used to draw arcs and circles.

### Letter and labelling convention



**Angle Notation:** three letters ABC  
This is the angle at B = 113°

**Line Notation:** two letters EC  
The line that joins E to C.

### Draw and measure line segments

Conversions 1cm = 10mm, 1m = 100cm

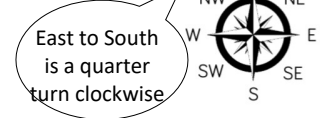
The line segment is 3.9cm  
Which is 39mm



AB is a **line segment** (part of the line)

Make sure the start of the line is at 0;

### Angles as measures of turn

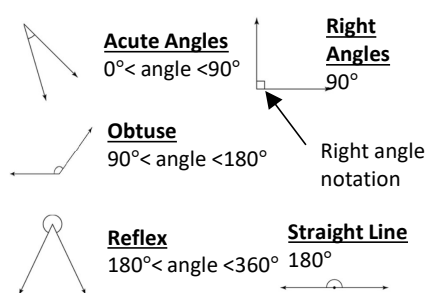


East to South is a quarter turn clockwise

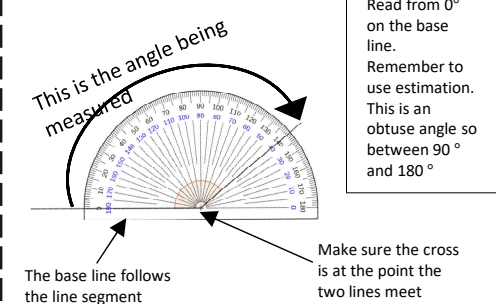


**Quarter Turn** 90° Clockwise  
**Half Turn** 180°  
**Three-quarter Turn** 270° Anti-Clockwise  
**Full Turn** 360°

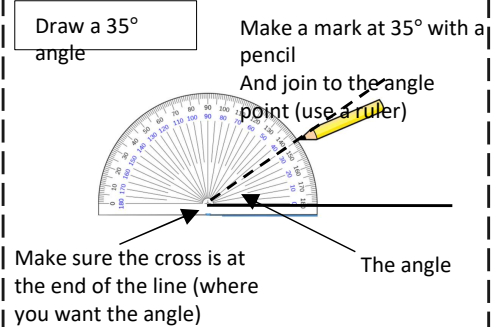
### Classify angles



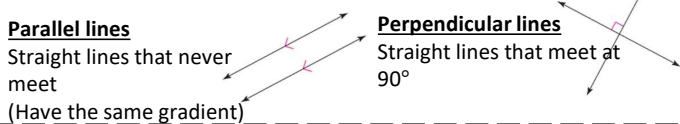
### Measure angles to 180°



### Draw angles up to 180°



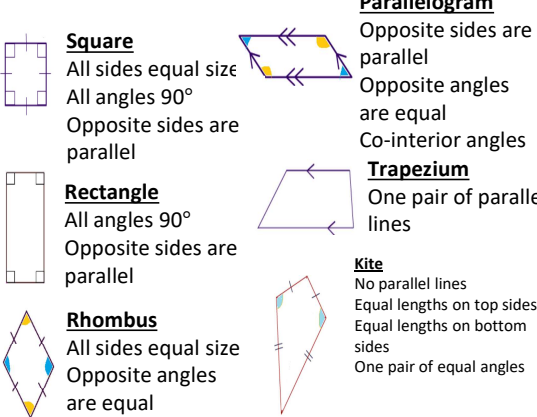
### Parallel and Perpendicular lines



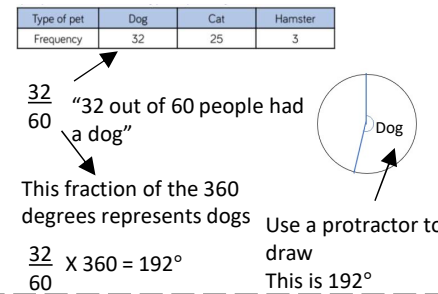
### Angles over 180°



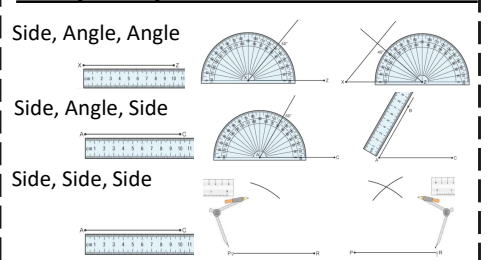
### Properties of Quadrilaterals



### Draw Pie Charts



### SAS, SSS, ASA constructions



### Polygons

3	- Triangle	5	- Pentagon	8	- Octagon
4	- Quadrilateral	6	- Hexagon	9	- Nonagon
		7	- Heptagon	10	- Decagon

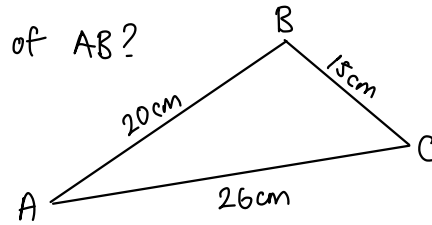
If all the sides and angles are the same, it is a **regular** polygon

**Question 1**

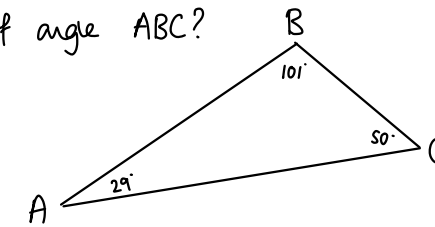
What is a polygon?

**Question 2**

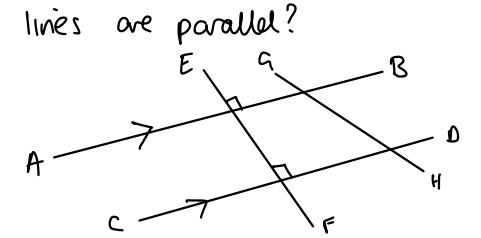
What is the length of AB?

**Question 3**

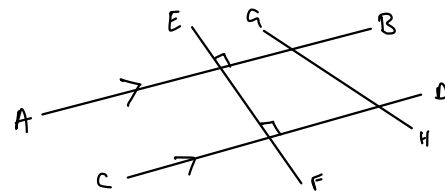
What is the size of angle ABC?

**Question 4**

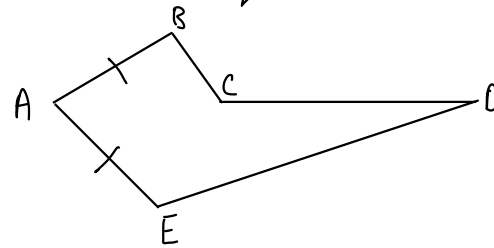
Which lines are parallel?

**Question 5**

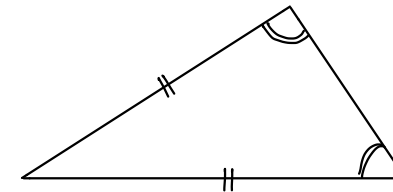
Which line is perpendicular to AB?

**Question 6**

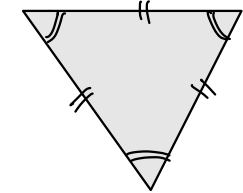
Which two lines are equal in length?

**Question 7**

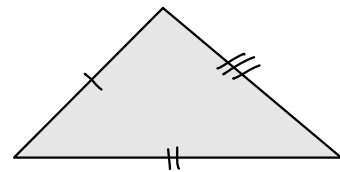
What type of triangle is this?

**Question 8**

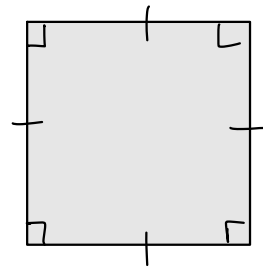
What type of triangle is this?

**Question 9**

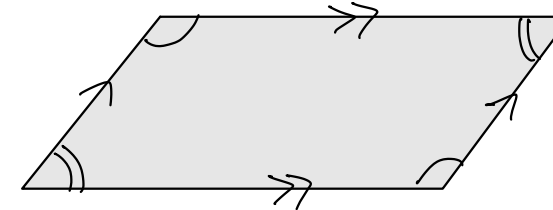
What type of triangle is this?

**Question 10**

Write down the mathematical name for this shape:

**Question 11**

Write down the mathematical name for this shape:

**Question 12**

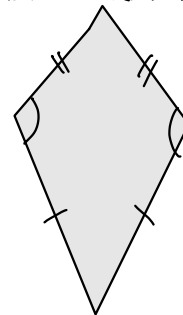
How many degrees are there in a full turn?

**Question 13**

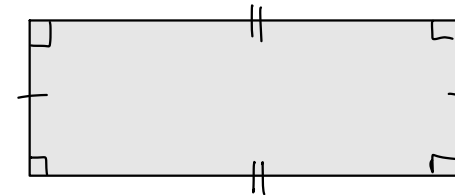
What angle is needed to represent 40 out of 60 on a pie chart?

**Question 14**

Write down the mathematical name for this shape:

**Question 15**

Write down the mathematical name for this shape:

**Question 16**

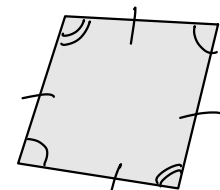
What angle is needed to represent 150 out of 240 on a pie chart?

**Question 17**

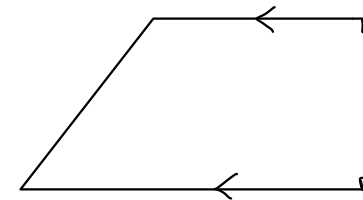
What angle is needed to represent 12 out of 72 on a pie chart?

**Question 18**

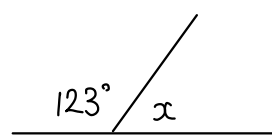
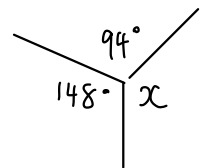
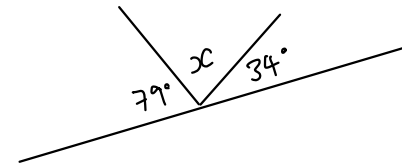
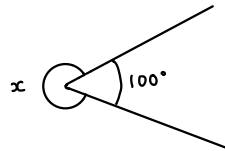
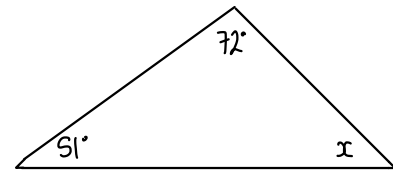
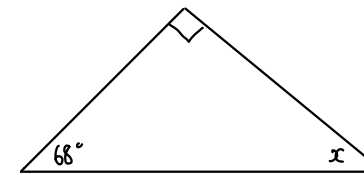
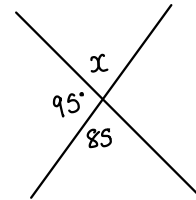
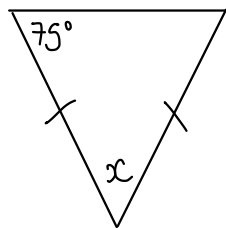
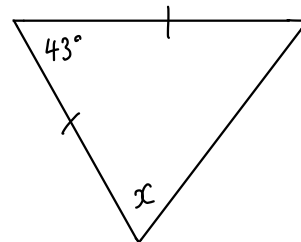
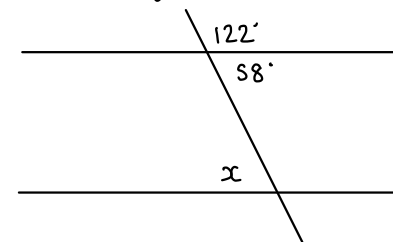
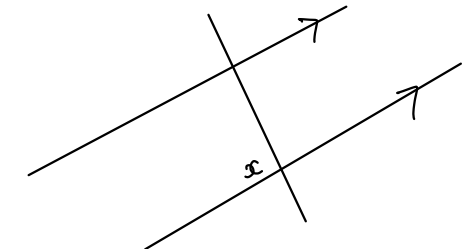
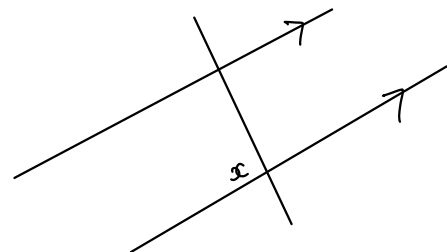
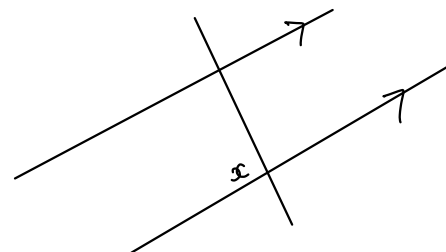
Write down the mathematical name for this shape:

**Question 19**

Write down the mathematical name for this shape:

**Question 20**

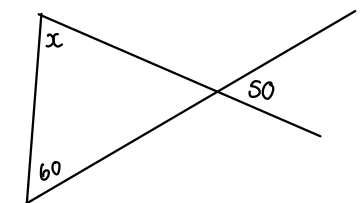
What angle is needed to represent 84 out of 100 on a pie chart?

**Question 1**Calculate the angle marked  $x$ **Question 2**Calculate the angle marked  $x$ **Question 3**Calculate the angle marked  $x$ **Question 4**Calculate the angle marked  $x$ **Question 5**Calculate the angle marked  $x$ **Question 6**Calculate the angle marked  $x$ **Question 7**Calculate the angle marked  $x$ **Question 8**Calculate the angle marked  $x$ **Question 9**Calculate the angle marked  $x$ **Question 10**Calculate the angle marked  $x$ **Question 11**Calculate the angle marked  $x$ **Question 12**Mark an angle that is co-interior to  $x$ **Question 13**Mark an angle that is alternate to  $x$ **Question 14**Mark an angle that is corresponding to  $x$ **Question 15**

Complete the sentence:

Co-interior angles sum to:

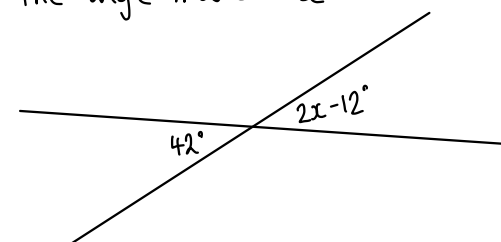
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**Question 16**Calculate the size of the angle marked  $x$ :**Question 17**

What is the interior angle sum of a pentagon?

**Question 18**

What is the size of each interior angle in a regular hexagon?

**Question 19**Calculate the angle marked  $x$ **Question 20**A regular polygon has an interior angle of  $172^\circ$ . How many sides does the polygon have?

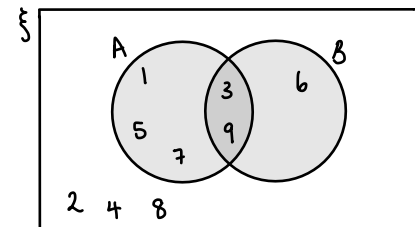
### Question 1

What is the probability of rolling a 6 on a regular, fair dice?

### Question 2

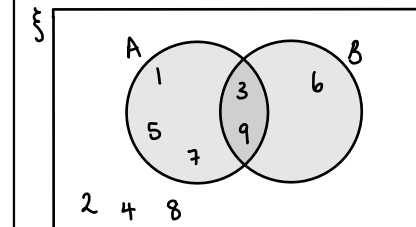
What is the probability of rolling an even number on a regular, fair dice?

### Question 3



Which numbers are elements of set A?

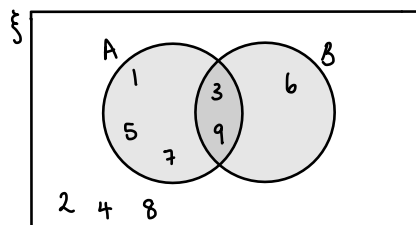
### Question 4



Which numbers are elements of set B?

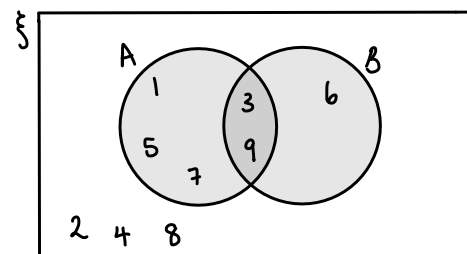
### Question 5

Which numbers are elements of set  $A \cap B$ ?



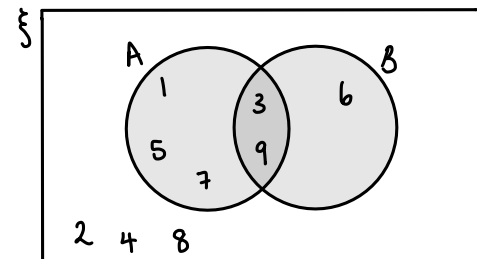
### Question 6

Which numbers are elements of set  $A \cup B$ ?



### Question 7

List the numbers in the complement of A.



### Question 9

$S = \{\text{numbers between 1 and 100 inclusive}\}$

$A = \{\text{Cube numbers}\}$

List the elements of set A

### Question 10

In a deck of cards, picking a red card and picking a King are mutually exclusive.

True or false?

### Question 11

In a deck of cards, picking a red card and picking a spade are mutually exclusive.

True or false?

### Question 12

In a regular deck of cards (not including Jokers), what is the probability of picking the Ace of Diamonds?

### Question 13

Describe the set in words:

$\{3, 6, 9, 12, 15\}$

### Question 14

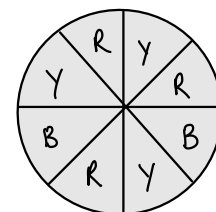
These letters are put into a hat:

M A T H E M A T I C S

Write the sample space for the outcomes

### Question 15

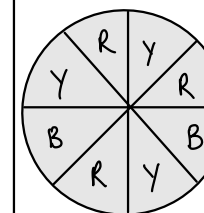
Mike spins the spinner. Each section is equally likely to be landed upon.



What is the probability of landing on red?

### Question 16

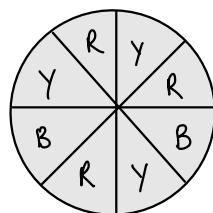
Mike spins the spinner. Each section is equally likely to be landed upon.



What is the probability of landing on blue?

### Question 17

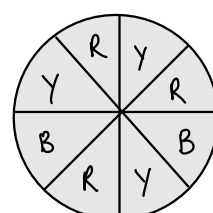
Mike spins the spinner. Each section is equally likely to be landed upon.



What is the probability of landing on red or yellow?

### Question 18

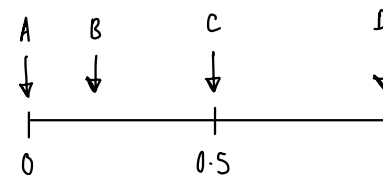
Mike spins the spinner. Each section is equally likely to be landed upon.



What is the probability of landing on green?

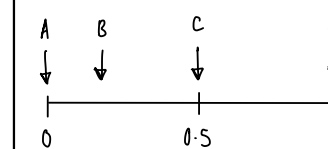
### Question 19

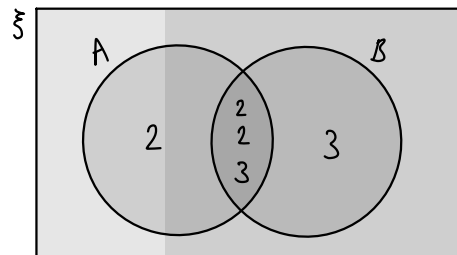
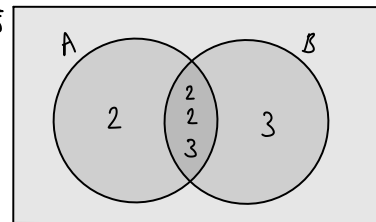
Which letter on the probability scale represents the probability of flipping a tail on a fair standard coin?



### Question 20

Which letter on the probability scale represents the probability of rolling a 4 on a standard dice?



<p><b>Question 1</b> List the first five multiples of 7</p>	<p><b>Question 2</b> List the first five multiples of 13</p>	<p><b>Question 3</b> List all of the factors of 35</p>	<p><b>Question 4</b> How many factors does a prime number have?</p>
<p><b>Question 5</b> What is the LCM of 8 and 10?</p>	<p><b>Question 6</b> What is the LCM of <math>2x</math> and <math>6x</math>?</p>	<p><b>Question 7</b> What is the HCF of 36 and 27?</p>	<p><b>Question 8</b> What is the HCF of 42 and 36?</p>
<p><b>Question 9</b> List the first ten prime numbers.</p>	<p><b>Question 10</b> List the first ten square numbers</p>	<p><b>Question 11</b> List the first ten cube numbers</p>	<p><b>Question 12</b> List the first ten triangle numbers</p>
<p><b>Question 13</b> By drawing a prime factor tree, express 80 as a product of prime factors</p>	<p><b>Question 14</b> By drawing a prime factor tree, express 450 as a product of prime factors</p>	<p><b>Question 15</b> <math>P = 3^2 \times 5^3 \times 7</math> <math>Q = 3 \times 5^4 \times 7^2</math>  What is the LCM of P and Q?</p>	<p><b>Question 16</b> <math>A = 2^2 \times 3^2 \times 5^2 \times 7</math> <math>B = 2^2 \times 3 \times 5^3</math>  What is the LCM of A and B?</p>
<p><b>Question 17</b> <math>P = 3^2 \times 5^3 \times 7</math> <math>Q = 3 \times 5^4 \times 7^2</math>  What is the HCF of P and Q?</p>	<p><b>Question 18</b> <math>A = 2^2 \times 3^2 \times 5^2 \times 7</math> <math>B = 2^2 \times 3 \times 5^3</math>  What is the HCF of A and B?</p>	<p><b>Question 19</b> What is the LCM of A and B?</p> 	<p><b>Question 20</b> What is the HCF of A and B?</p> 

## Geometric reasoning

### What do I need to be able to do?

By the end of this unit you should be able to:

- Understand/use the sum of angles at a point
- Understand/use the sum of angles on a straight line
- Understand/use equality of vertically opposite angles
- Know and apply the sum of angles in a triangle
- Know and apply the sum of angles in a quadrilateral

### Keywords

**Vertically Opposite:** angles formed when two or more straight lines cross at a point.

**Interior Angles:** angles inside the shape

**Sum:** total, add all the interior angles together

**Convex Quadrilateral:** a four-sided polygon where every interior angle is less than  $180^\circ$

**Concave Quadrilateral:** a four-sided polygon where one interior angle exceeds  $180^\circ$

**Polygon:** A 2D shape made with straight lines

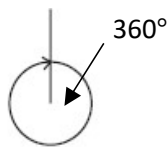
**Scalene triangle:** a triangle with all different sides and angles

**Isosceles triangle:** a triangle with two angles the same size and two sides the same size

**Right-angled triangle:** a triangle with a right angle

### Sum of angles at a point

The sum of angles around a point is  $360^\circ$



Find angle BOE

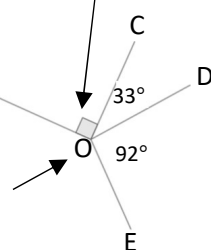
$$90^\circ + 33^\circ + 92^\circ = 205^\circ$$

$$360^\circ - 205^\circ = \text{BOE}$$

$$\text{BOE} = 155^\circ$$

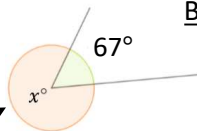
Angle notation –

$$90^\circ$$



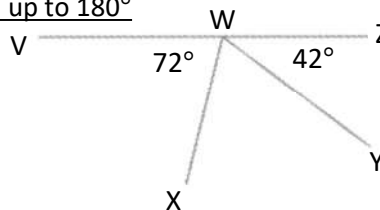
Angle notation – find this missing angle

$$360^\circ - 67^\circ = 293^\circ$$



### Sum of angles on a straight line

Adjacent angles that share a common point on a line add up to  $180^\circ$

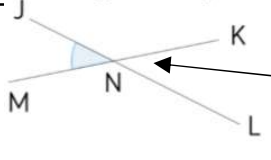


Find angle XWY

$$72^\circ + 42^\circ = 114^\circ$$

$$180^\circ - 114^\circ = 66^\circ$$

### Vertically opposite angles

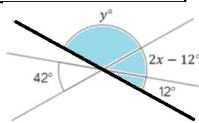


Angle JNM is vertically opposite to angle KNL

$$\text{JNM} = \text{KNL}$$

Vertically opposite angles are the same

Other angle rules still apply. Look for straight line sums and angles around a



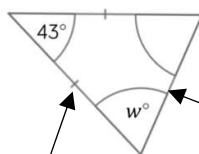
Form equations with information from diagrams:

$$2x - 12 = 42$$

$$2x = 54$$

$$x = 27^\circ$$

### Sum of angles in triangles

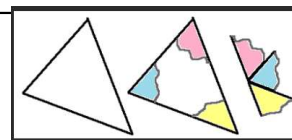


Look at triangle notation. This indicates an isosceles triangle  
 $\therefore 180 - 43 = 137$   
 $137 \div 2 = 68.5^\circ$

The two base angles will be the same size

A triangle can only have ONE right angle

Sum of interior angles in a triangle =  $180^\circ$



Have a go!  
Tearing the corners from triangles forms a straight line which is therefore  $180^\circ$

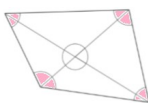
### Sum of angles in quadrilaterals



Convex Quadrilateral

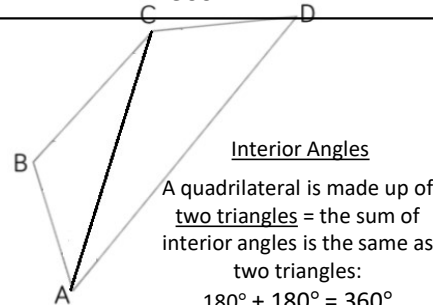


Concave Quadrilateral



Interior angles are those that make up the perimeter (outline) of the shape

Sum of interior angles in a quadrilateral =  $360^\circ$

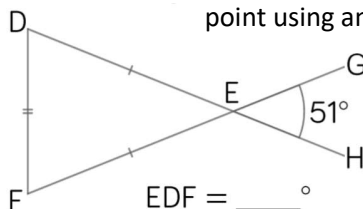


Interior Angles

A quadrilateral is made up of two triangles = the sum of interior angles is the same as two triangles:  
 $180^\circ + 180^\circ = 360^\circ$

### Angle Problems

Split up the problem into chunks and explain your reasoning at each point using angle notation



$$\text{EDF} = \underline{\quad}^\circ$$

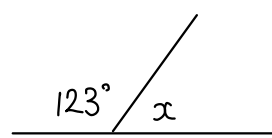
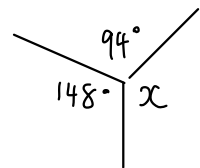
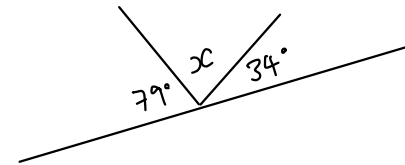
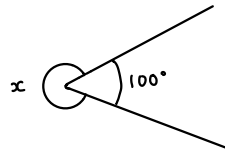
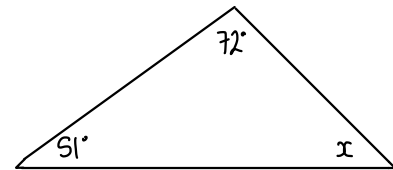
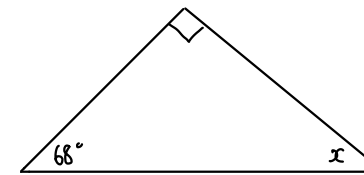
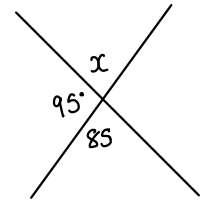
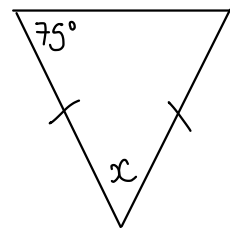
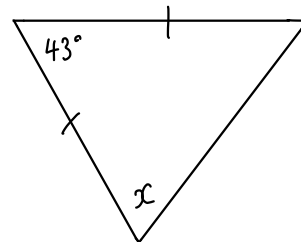
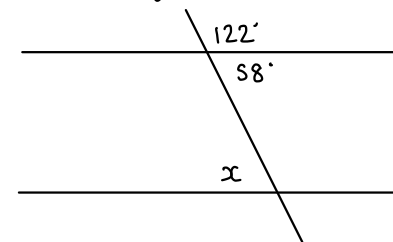
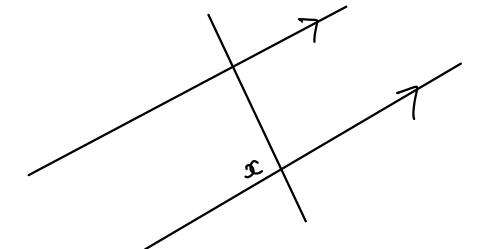
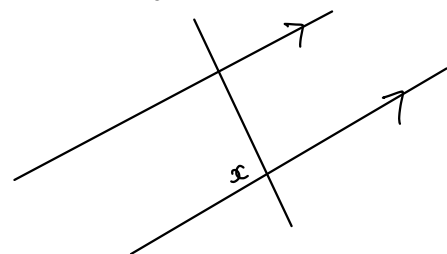
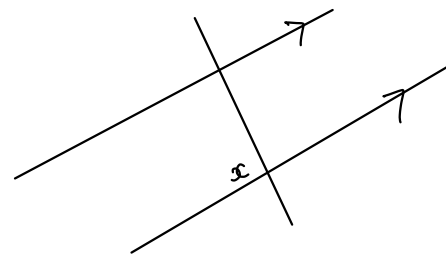
1. Angle DEF =  $51^\circ$  because it is a vertically opposite angle DEF = GEH

2. Triangle DEF is isosceles (triangle notation)  $\therefore \text{EDF} = \text{EFD}$  and the sum of interior angles is  $180^\circ$   
 $180^\circ - 51^\circ = 129^\circ$      $129^\circ \div 2 = 64.5^\circ$

3. Angle EDF =  $64.5^\circ$

Keep working out clear and notes together

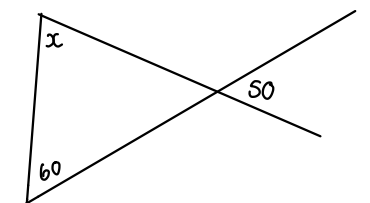


**Question 1**Calculate the angle marked  $x$ **Question 2**Calculate the angle marked  $x$ **Question 3**Calculate the angle marked  $x$ **Question 4**Calculate the angle marked  $x$ **Question 5**Calculate the angle marked  $x$ **Question 6**Calculate the angle marked  $x$ **Question 7**Calculate the angle marked  $x$ **Question 8**Calculate the angle marked  $x$ **Question 9**Calculate the angle marked  $x$ **Question 10**Calculate the angle marked  $x$ **Question 11**Calculate the angle marked  $x$ **Question 12**Mark an angle that is co-interior to  $x$ **Question 13**Mark an angle that is alternate to  $x$ **Question 14**Mark an angle that is corresponding to  $x$ **Question 15**

Complete the sentence:

Co-interior angles sum to:

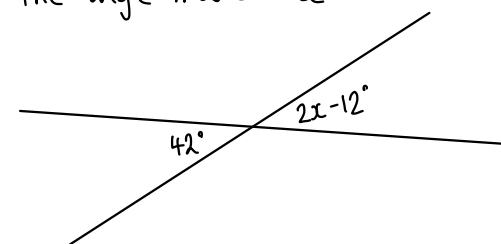
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**Question 16**Calculate the size of the angle marked  $x$ :**Question 17**

What is the interior angle sum of a pentagon?

**Question 18**

What is the size of each interior angle in a regular hexagon?

**Question 19**Calculate the angle marked  $x$ **Question 20**A regular polygon has an interior angle of  $172^\circ$ . How many sides does the polygon have?

# Year 7 – reasoning with number



## Developing number sense

### What do I need to be able to do?

By the end of this unit you should be able to:

- Know and use mental addition/ subtraction
- Know and use mental multiplication/ division
- Know and use mental arithmetic for decimals
- Know and use mental arithmetic for fractions
- Use factors to simplify calculations
- Use estimation to check mental calculations
- Use number facts
- Use algebraic facts

### Keywords

- Commutative:** changing the order of the operations does not change the result
- Associative:** when you add or multiply you can do so regardless of how the numbers are grouped
- Dividend:** the number being divided
- Divisor:** the number we divide by.
- Expression:** a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
- Equation:** a mathematical statement that two things are equal
- Quotient:** the result of a division

### Mental methods for addition/ subtraction

Addition is commutative

Subtraction the order has to stay the same



$$6 + 3 = 3 + 6$$

The order of addition does not change the result

$$360 - 147 = 360 - 100 - 40 - 7$$

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction

### Mental methods for multiplication/ division

Multiplication is commutative

Partitioning can help multiplication



$$2 \times 4 = 4 \times 2$$

The order of multiplication does not change the result

$$\begin{aligned} 24 \times 6 &= 20 \times 6 + 4 \times 6 \\ &= 120 + 24 \\ &= 144 \end{aligned}$$

Division is not associative

Chunking the division can help  $4000 \div 25$   
"How many 25's in 100" then how many chunks of that in 4000.

### Mental methods for decimals

Multiplying by a decimal  $< 1$  will make the original value smaller e.g  $\times 0.1 = \div 10$

Methods for multiplication  $1.2 \times 0.03$

$$\begin{array}{l} 1.2 \times 3 = 3.6 \\ 1.2 \times 0.3 = 0.36 \\ 1.2 \times 0.03 = 0.036 \end{array} \quad \begin{array}{l} 1.2 \times 3 = 3.6 \\ \div 10 \quad \div 100 \quad \div 1000 \\ 1.2 \times 0.3 = 0.36 \\ 1.2 \times 0.03 = 0.036 \end{array}$$

Methods for division  $1.5 \div 0.05$

Multiply by powers of 10 until the divisor becomes an integer

$$\begin{array}{l} 1.5 \div 0.05 \\ \times 100 \quad \times 100 \\ 150 \div 5 = 30 \end{array}$$

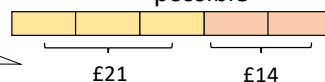
Methods for addition

$$\begin{array}{l} 2 + 2 = 4 \\ 0.3 + 0.4 = 0.7 \\ 4 + 0.7 = 4.7 \end{array}$$

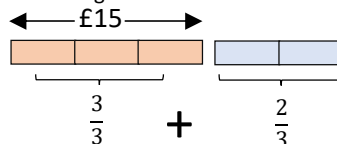
### Mental methods for fractions

Use bar models where possible

I've spent  $\frac{2}{5}$  of my money. I have £21 left



How much did they have to begin with?



What is  $\frac{5}{3}$  of £15?

### Using factors to simplify calculations

$$30 \times 16$$

$$10 \times 3 \times 4 \times 4$$

$$10 \times 3 \times 2 \times 8$$

$$2 \times 5 \times 3 \times 2 \times 2 \times 2 \times 2$$

$$16 \times 10 \times 3$$

Multiplication is commutative  
Factors can be multiplied in any order

### Estimation

Estimations are useful – especially when using fractions and decimals to check if your solution is possible.

Most estimations round to 1 significant figure

$$210 + 899 < 1200$$

This is true because even if both numbers were rounded up, they would reach 300 + 900.

The correct estimation would be  $200 + 900 = 1100$ .

### Number facts

Use  $124 \times 5 = 620$

For multiplication, each value that is multiplied or divided by powers of 10 needs to happen to the result

$$620 \div 12.4 = 50$$

For division you must consider the impact of the divisor becoming smaller or bigger.

Smaller – the answer will be bigger (It is being shared into less parts)  
Bigger – the answer will be smaller (It is being shared into more parts)

### Algebraic facts

$$2a + 2b = 10 \quad \text{Everything} \times 2$$

$$0.1a + 0.1b = 0.5 \quad \text{Everything} \div 10$$

$$a + b = 5$$

$$a + b + 2 = 7 \quad \text{Add 2 to the total}$$

The unknown quantity isn't changing but the variables change what is done to give the result.

# YEAR 7 — REASONING WITH NUMBER

## Developing number sense

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Know and use mental addition/ subtraction
- Know and use mental multiplication/ division
- Know and use mental arithmetic for decimals
- Know and use mental arithmetic for fractions
- Use factors to simplify calculations
- Use estimation to check mental calculations
- Use number facts
- Use algebraic facts

### Keywords

- Commutative:** changing the order of the operations does not change the result
- Associative:** when you add or multiply you can do so regardless of how the numbers are grouped
- Dividend:** the number being divided
- Divisor:** the number we divide by
- Expression:** a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
- Equation:** a mathematical statement that two things are equal
- Quotient:** the result of a division

### Mental methods for addition/ subtraction

Addition is commutative



$$6 + 3 = 3 + 6$$

The order of addition does not change the result

Subtraction the order has to stay the same

$$360 - 147 = 360 - 100 - 40 - 7$$

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction

### Mental methods for multiplication/ division

Multiplication is commutative



$$2 \times 4 = 4 \times 2$$

The order of multiplication does not change the result

Partitioning can help multiplication

$$\begin{aligned} 24 \times 6 &= 20 \times 6 + 4 \times 6 \\ &= 120 + 24 \\ &= 144 \end{aligned}$$

Division is not associative

Chunking the division can help  $4000 \div 25$   
"How many 25's in 100" then how many chunks of that in 4000.

### Mental methods for decimals

Multiplying by a decimal  $< 1$  will make the original value smaller e.g.  $0.1 = \div 10$

Methods for multiplication  $12 \times 0.03$

$$\begin{array}{l} 12 \times 3 = 36 \\ 12 \times 3 = 36 \\ 12 \times 0.3 = 3.6 \\ 12 \times 0.03 = 0.36 \end{array} \quad \begin{array}{l} 12 \times 3 = 36 \\ +10 \downarrow +100 \downarrow +1000 \downarrow \\ 12 \times 0.03 = 0.36 \end{array}$$

Methods for division  $15 \div 0.05$

Multiply by powers of 10 until the divisor becomes an integer

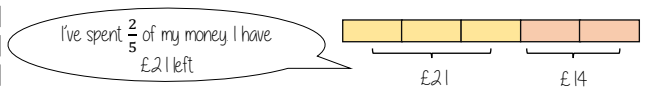
$$\begin{array}{l} 1.5 \div 0.05 \\ \times 100 \downarrow \quad \times 100 \downarrow \\ 150 \div 5 = 30 \end{array}$$

Methods for addition  $2.3 + 2.4$

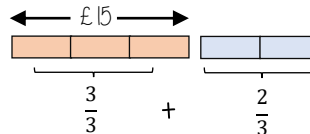
$$\begin{array}{l} 2 + 2 = 4 \\ 0.3 + 0.4 = 0.7 \\ 4 + 0.7 = 4.7 \end{array}$$

### Mental methods for fractions

Use bar models where possible



How much did they have to begin with?



What is  $\frac{5}{3}$  of £15?

### Using factors to simplify calculations

$$30 \times 16$$

$$10 \times 3 \times 4 \times 4$$

$$10 \times 3 \times 2 \times 8$$

$$2 \times 5 \times 3 \times 2 \times 2 \times 2$$

$$16 \times 10 \times 3$$

Multiplication is commutative  
Factors can be multiplied in any order

### Estimation

Estimations are useful — especially when using fractions and decimals to check if your solution is possible.

Most estimations round to 1 significant figure

Estimations are useful — especially when using fractions and decimals to check if your solution is possible.

$$210 + 899 < 1200$$

This is true because even if both numbers were rounded up, they would reach  $300 + 900$ .

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Use  $124 \times 5 = 620$

For multiplication, each value that is multiplied or divided by powers of 10 needs to happen to the result

$$620 \div 124 = 50$$

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Smaller — the answer will be bigger (It is being shared into less parts)  
Bigger — the answer will be smaller (It is being shared into more parts)

### Algebraic facts

$$2a + 2b = 10 \quad \text{Everything } \times 2$$

$$0.1a + 0.1b = 0.5$$

Everything  $\div 10$

$$a + b = 5$$

Add 2 to the total

$$a + b + 2 = 7$$

The unknown quantity isn't changing but the variables change what is done to give the result

# Year 7 – reasoning with number



@whisto\_maths

## Prime numbers and Proof

### What do I need to be able to do?

By the end of this unit you should be able to:

- Find and use multiples
- Identify factors of numbers and expressions
- Recognise and identify prime numbers
- Recognise square and triangular numbers
- Find common factors including HCF
- Find common multiples including LCM

### Keywords

**Multiples:** found by multiplying any number by positive integers

**Factor:** integers that multiply together to get another number.

**Prime:** an integer with only 2 factors.

**Conjecture:** a statement that might be true (based on reasoning) but is not proven.

**Counterexample:** a special type of example that disproves a statement.

**Expression:** a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

**HCF:** highest common factor (biggest factor two or more numbers share)

**LCM:** lowest common multiple (the first time the times table of two or more numbers match)

### Multiples

The "times table" of a given number  
All the numbers in this lists below are multiples of 3.

3, 6, 9, 12, 15...

$3x, 6x, 9x \dots$

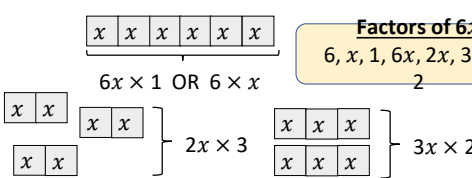
This list continues and doesn't end  
**Non example of a multiple**  
4.5 is not a multiple of 3 because it is  $3 \times 1.5$   
1.5  
Not an integer

$x$  could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

### Factors

Arrays can help represent factors  
 $5 \times 2$  or  $2 \times 5$   
**Factors of 10**  
1, 2, 5, 10  
The number itself is always a factor

Factors and expressions



### Prime numbers

2

- Integer
- Only has 2 factors
- and itself

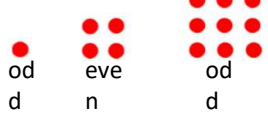
The first prime number  
The only even prime number

Learn or how-to quick recall...

2, 3, 5, 7, 11, 13, 17, 19, 23, 29...

### Square and triangular numbers

#### Square numbers



Representations are useful to understand a square number  $n^2$   
**1, 4, 9, 16, 25, 36, 49, 64 ...**

#### Triangular numbers

Representations are useful – an extra counter is added to each new row  
Add two consecutive triangular numbers and get a square number  
**1, 3, 6, 10, 15, 21, 28, 36, 45...**

### Common factors and HCF

1 is a common factor of all numbers

Common factors are factors two or more numbers share

#### HCF – Highest common factor

**HCF of 18 and 30**

**18** 1, 2, 3, 6, 9,  
18  
**30** 1, 2, 3, 5, 6, 10, 15,  
30

Common factors (factors of both numbers)  
1, 2, 3, 6

**HCF = 6**

6 is the biggest factor they share

### Common multiples and LCM

#### LCM – Lowest common multiple

**LCM of 9 and 12**

**9** 9, 18, 27, 36, 45,  
54  
**12** 12, 24, 36, 48, 60

**LCM = 36**  
The first time their multiples match



#### Comparing fractions

Compare fractions using a LCM denominator

$\frac{3}{5}$  and  $\frac{7}{10}$

$\frac{6}{10}$  and  $\frac{7}{10}$

### Conjectures and counterexamples

#### Conjecture

1, 2, 4...  
The numbers in the sequence are doubling each time.

A pattern that is noticed for many cases

#### Counterexamples

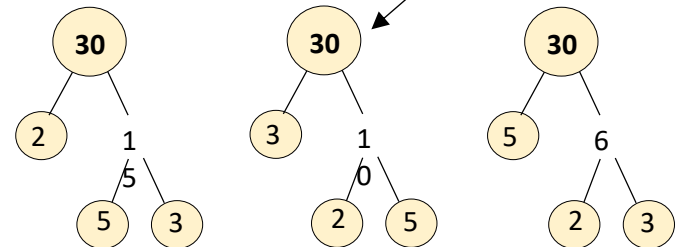


This sequence isn't doubling it is adding 2 each time

Only **one** counterexample is needed to disprove a conjecture

### Product of prime factors

Multiplication part-whole models



All three prime factor trees represent the same decomposition

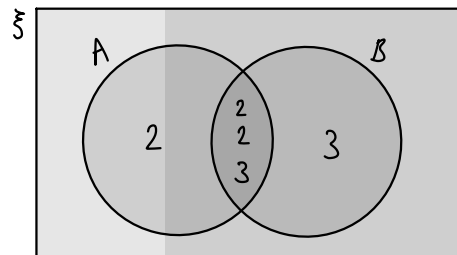
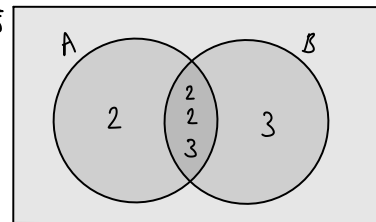
Multiplication is commutative

**$30 = 2 \times 3 \times 5$**

Multiplication of prime factors

Using prime factors for predictions

e.g. **60**  $30 \times 2$   $2 \times 3 \times 5 \times 2$   
**150**  $30 \times 5$   $2 \times 3 \times 5 \times 5$

<p><b>Question 1</b> List the first five multiples of 7</p>	<p><b>Question 2</b> List the first five multiples of 13</p>	<p><b>Question 3</b> List all of the factors of 35</p>	<p><b>Question 4</b> How many factors does a prime number have?</p>
<p><b>Question 5</b> What is the LCM of 8 and 10?</p>	<p><b>Question 6</b> What is the LCM of <math>2x</math> and <math>6x</math>?</p>	<p><b>Question 7</b> What is the HCF of 36 and 27?</p>	<p><b>Question 8</b> What is the HCF of 42 and 36?</p>
<p><b>Question 9</b> List the first ten prime numbers.</p>	<p><b>Question 10</b> List the first ten square numbers</p>	<p><b>Question 11</b> List the first ten cube numbers</p>	<p><b>Question 12</b> List the first ten triangle numbers</p>
<p><b>Question 13</b> By drawing a prime factor tree, express 80 as a product of prime factors</p>	<p><b>Question 14</b> By drawing a prime factor tree, express 450 as a product of prime factors</p>	<p><b>Question 15</b> <math>P = 3^2 \times 5^3 \times 7</math> <math>Q = 3 \times 5^4 \times 7^2</math>  What is the LCM of P and Q?</p>	<p><b>Question 16</b> <math>A = 2^2 \times 3^2 \times 5^2 \times 7</math> <math>B = 2^2 \times 3 \times 5^3</math>  What is the LCM of A and B?</p>
<p><b>Question 17</b> <math>P = 3^2 \times 5^3 \times 7</math> <math>Q = 3 \times 5^4 \times 7^2</math>  What is the HCF of P and Q?</p>	<p><b>Question 18</b> <math>A = 2^2 \times 3^2 \times 5^2 \times 7</math> <math>B = 2^2 \times 3 \times 5^3</math>  What is the HCF of A and B?</p>	<p><b>Question 19</b> What is the LCM of A and B?</p> 	<p><b>Question 20</b> What is the HCF of A and B?</p> 

## Sets and probability

### What do I need to be able to do?

By the end of this unit you should be able to:

- Identify and represent sets
- Interpret and create Venn diagrams
- Understand and use the intersection of sets
- Understand and use the union of sets
- Generate sample spaces for single events
- Calculate the probability of a single event
- Understand and use the probability scale

### Keywords

**Set:** collection of things

**Element:** each item in a set is called an element

**Intersection:** the overlapping part of a Venn diagram (AND  $\cap$ )

**Union:** two ellipses that join (OR  $\cup$ )

**Mutually Exclusive:** events that do not occur at the same time

**Probability:** likelihood of an event happening

**Bias:** a built-in error that makes all values wrong (unequal) by a certain amount, e.g. a weighted dice

**Fair:** there is zero bias, and all outcomes have an equal likelihood

**Random:** something happens by chance and is unable to be predicted.

### Identify and represent sets

The **universal set** has this symbol  $\xi$  – this means **EVERYTHING** in the Venn diagram is in this set

A set is a collection of things – you write sets inside curly brackets { }

$\xi = \{\text{the numbers between 1 and 50 inclusive}\}$

My sets can include every number between 1 and 50 including those numbers

$A = \{\text{Square numbers}\}$

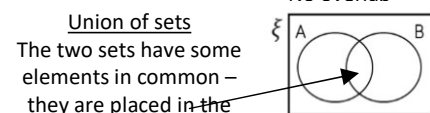
$A = \{1, 4, 9, 16, 25, 36, 49\}$

All the numbers in set A are square number and between 1 and 50

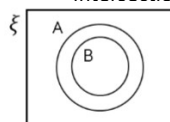
### Interpret and create Venn diagrams



**Mutually exclusive sets**  
The two sets have nothing in common  
No overlap



**Union of sets**  
The two sets have some elements in common – they are placed in the intersection



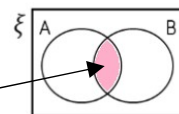
**Subset**  
All of set B is also in Set A so the ellipse fits inside the set.

**The box**

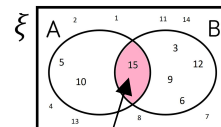
Around the outside of every Venn diagram will be a box. If an element is not part of any set it is placed outside an ellipse but inside the box

### Intersection of sets

Elements in the intersection are in set A AND set B  
The notation for this is  $A \cap B$



$\xi = \{\text{the numbers between 1 and 15 inclusive}\}$   
 $A = \{\text{Multiples of 3}\}$   
 $B = \{\text{Multiples of 5}\}$

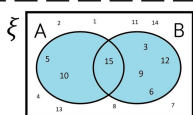


The element in  $A \cap B$  is 15

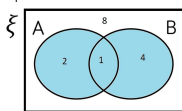
In this example there is only one number that is both a multiple of 3 and a multiple of 5 between 1 and 15

### Union of sets

Elements in the union could be in set A OR set B



The notation for this is  $A \cup B$



This Venn shows the **number of elements** in each set

$\xi = \{\text{the numbers between 1 and 15 inclusive}\}$   
 $A = \{\text{Multiples of 5}\}$   $B = \{\text{Multiples of 3}\}$   
The elements in  $A \cup B$  are

5, 10, 15, 3, 9, 6, 12  
There are 7 elements that are either a multiple of 5 OR a multiple of 3 between 1 and 15

### Sample space – for single events



A sample space for rolling a six-sided dice is  
 $S = \{1, 2, 3, 4, 5, 6\}$



A sample space for this spinner is  
 $S = \{\text{Pink, Blue, Yellow}\}$

You only need to write each element once in a sample space diagram

- A Sample space represents a possible outcome from an event
- They can be interpreted in a variety of ways because they do not tell you the probability

### Probability of a single event

Probability =  $\frac{\text{number of times event happens}}{\text{total number of possible outcomes}}$



$P(\text{Blue}) = \frac{4}{10}$  There are 4 blue sectors

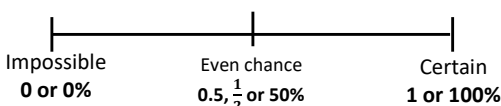
Probability notation  $P(\text{event}) = \frac{2}{5}$  There are 10 sectors overall

Probability can be a fraction, decimal or percentage value

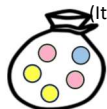
$$\frac{4}{10} = \frac{40}{100} = 0.40 = 40\%$$

Probability is always a value between 0 and 1

### The probability scale



The more likely an event the further up the probability it will be in comparison to another event (It will have a probability closer to 1)



There are 2 pink and 2 yellow balls, so they have the same probability

There are 5 possible outcomes  
So 5 intervals on this scale, each interval value is  $\frac{1}{5}$

### Sum of probabilities

Probability is always a value between 0 and 1



The probability of getting a blue ball is  $\frac{1}{5}$

$\therefore$  The probability of **NOT** getting a blue ball is  $\frac{4}{5}$

The sum of the probabilities is 1

The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	

$$P(\text{white chocolate}) = 1 - 0.15 - 0.35 = 0.5$$





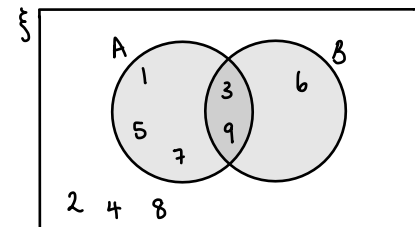
### Question 1

What is the probability of rolling a 6 on a regular, fair dice?

### Question 2

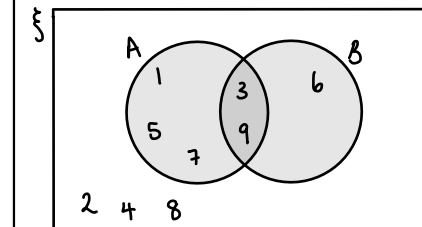
What is the probability of rolling an even number on a regular, fair dice?

### Question 3



Which numbers are elements of set A?

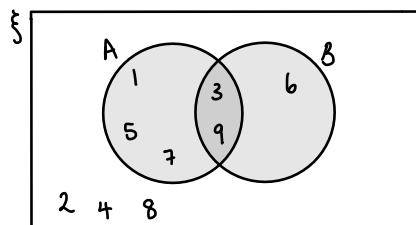
### Question 4



Which numbers are elements of set B?

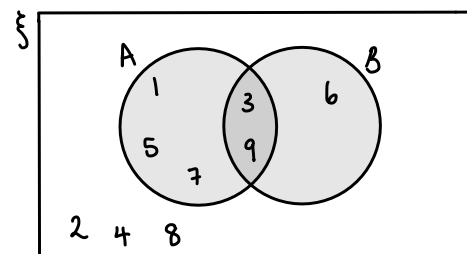
### Question 5

Which numbers are elements of set  $A \cap B$ ?



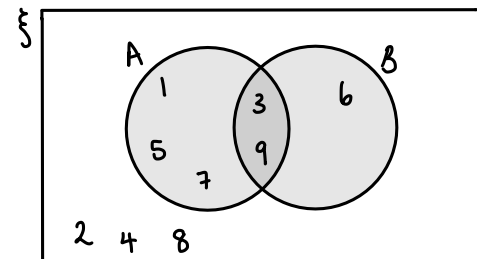
### Question 6

Which numbers are elements of set  $A \cup B$ ?



### Question 7

List the numbers in the complement of A.



### Question 8

The probability of picking a yellow card is  $\frac{1}{4}$ . What is the probability of not getting a yellow card?

### Question 9

$S = \{\text{numbers between 1 and 100 inclusive}\}$

$A = \{\text{Cube numbers}\}$

List the elements of set A

### Question 10

In a deck of cards, picking a red card and picking a King are mutually exclusive.

True or false?

### Question 11

In a deck of cards, picking a red card and picking a spade are mutually exclusive.

True or false?

### Question 12

In a regular deck of cards (not including Jokers), what is the probability of picking the Ace of Diamonds?

### Question 13

Describe the set in words:

$\{3, 6, 9, 12, 15\}$

### Question 14

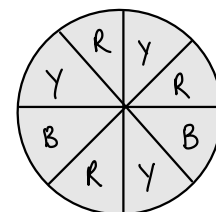
These letters are put into a hat:

M A T H E M A T I C S

Write the sample space for the outcomes

### Question 15

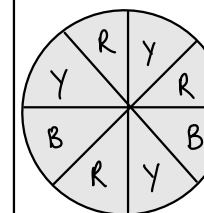
Mike spins the spinner. Each section is equally likely to be landed upon.



What is the probability of landing on red?

### Question 16

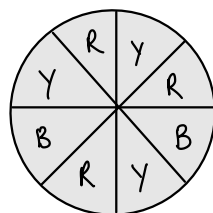
Mike spins the spinner. Each section is equally likely to be landed upon.



What is the probability of landing on blue?

### Question 17

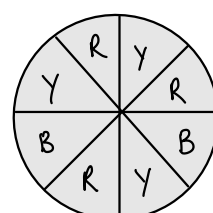
Mike spins the spinner. Each section is equally likely to be landed upon.



What is the probability of landing on red or yellow?

### Question 18

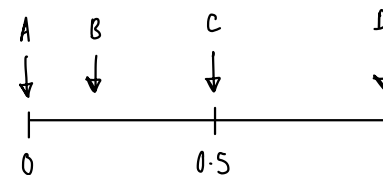
Mike spins the spinner. Each section is equally likely to be landed upon.



What is the probability of landing on green?

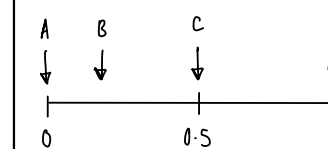
### Question 19

Which letter on the probability scale represents the probability of flipping a tail on a fair standard coin?



### Question 20

Which letter on the probability scale represents the probability of rolling a 4 on a standard dice?



# Year 8 - Proportional reasoning...



# Multiplicative Change

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems and explain direct proportion
- Use conversion graphs to make statements, comparisons and form conclusions.
- Understand and use scale factors for length

## Keywords

- Proportion:** a statement that links two ratios
- Variable:** a part that the value can be changed
- Axes:** horizontal and vertical lines that a graph is plotted around
- Approximation:** an estimate for a value
- Scale Factor:** the multiple that increases/ decreases a shape in size
- Currency:** the system of money used in a particular country
- Conversion:** the process of changing one variable to another
- Scale:** the comparison of something drawn to its actual size.

## Direct Proportion

As one variable changes the other changes at the same rate.

4 cans of pop = £2.40

4 cans of pop = £2.40  
 2 cans of pop = £1.20

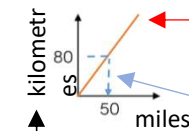
This multiplier is the same  
 In the same way that this would be for ratio

This is a multiplicative change  
 4 cans of pop = £2.40  
 12 cans of pop = £7.20

Sometimes this is easiest if you work out how much one unit is worth first  
 e.g. 1 can of pop = £0.60

## Conversion Graphs

Compare two variables



Labelling of both axes is vital

This is always a straight line because as one variable increases so does the other at the same rate

To make conversions between units you need to find the point to compare – then find the associated point by using your graph. Using a ruler helps for accuracy. Showing your conversion lines help as a “check” for solutions

## Conversion between currencies

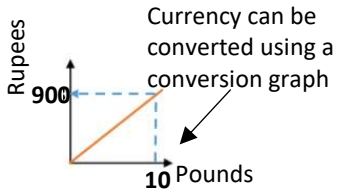


£1 = 90 Rupees

Currency is directly proportional

For every £1 I have 90 Rupees

£1 = 90 Rupees  
 £10 = 900 Rupees



Currency can be converted using a conversion graph

Convert 630 Rupees into Pounds

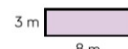
£1 = 90 Rupees  
 £7 = 630 Rupees  
 630 ÷ 90 = 7

## Ratio between similar shapes



Angles in similar shapes do not change.  
 e.g. if a triangle gets bigger the angles can not go above 180°

The two rectangles are similar.



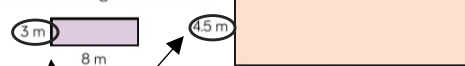
Corresponding sides

3m : 4.5m  
 1m : 1.5m  
 8m : 12m  
 1m : 1.5m

Note: Simplify to the same ratio

## Understand Scale Factor

The two rectangles are similar.



$$3 \times 1.5 = 4.5$$

This is a multiplicative change

Use corresponding sides to calculate a scale factor

Missing length  
 $8 \times 1.5 = 12m$

Scale factor can also be calculated by:

Small corresponding side  $\times$  SF = Big corresponding side  
 Big corresponding side  $\div$  SF = Small corresponding side

## Draw and interpret scale diagrams

A picture of a car is drawn with a scale of 1:30

For every 1cm on my image is 30cm in real life

The car image is 10cm

Image : Real life  
 1cm : 30cm  
 10cm : 300cm

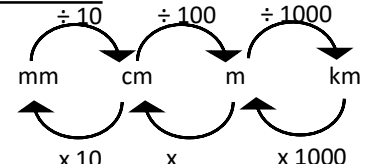


The car in real life is 210cm

Image : Real life  
 1cm : 30cm  
 7cm : 210cm



## Interpret maps with scale factors



1 cm : 250 m

1 cm : 250m

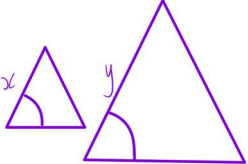
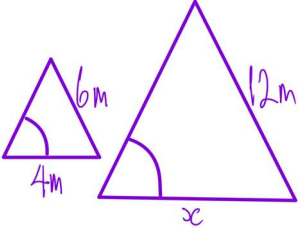
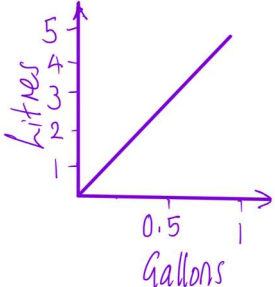
1 cm : 25000cm

250 x 100 = 25000  
 Ratios need to be in the same units

For every 1cm on my map is 25000cm in real life.





<p>Q1 - If 4 tickets cost £15.80 how much will 8 tickets cost?</p>	<p>Q2 - Given £1 is 25 Mexican Pesos, convert £50 into Mexican Pesos</p>	<p>Q3 – What is the scale factor to use on a map with a scale of 1cm = 25km?</p>	<p>Q4 - If 8 stamps cost £18.00, how much will 5 stamps cost?</p>	<p>Q5 – is it angles or side lengths that stay the same in similar shapes?</p> 
<p>Q6 – Given that £1 = €1.28 how many Euros is £25?</p>	<p>Q7 – An accurate drawing of a T-Rex fossil skeleton has the skull at 3cm long. If the scale of the drawing is 1:50, how long is the actual skull fossil?</p>	<p>Q8 - If 9 pencils cost £31.50 how much will 11 pencils cost?</p>	<p>Q9 – The straight-line distance between Manchester and Chester is 27 miles. What would the measurement between the two cities be on a map with a scale of 1cm = 3miles?</p>	<p>Q10 – If £1= \$1.08, how many pounds will I get for \$250, to the nearest penny?</p>
<p>Q11 - If 9 biscuits cost £13.50 how much will 6 biscuits cost?</p>	<p>Q12 – The two triangles are similar. What is x?</p> 	<p>Q13 – Estimate how many litres are in half a gallon.</p> 	<p>Q14 – £1 is worth \$1.278 dollars at today's exchange rate. How many £s will I get if I convert \$400?</p>	<p>Q15 – Two similar shapes have corresponding sides of 2cm and 9cm. Another side on the smaller shape is 7cm. What is the length of the corresponding side on the larger shape?</p>
<p>Q16 – Two similar shapes have corresponding sides of 2cm and 9cm. What is the scale factor?</p>	<p>Q17 – 5 miles is equal to 8km. How many miles is a 30k race (to 1 decimal place)?</p>	<p>Q18 – on a map with a scale of 1:10,000, a road is 4.5cm long. How long is the real road in km?</p>	<p>Q19 - If 11 sweets cost £13.75 how much will 3 sweets cost?</p>	<p>Q20 – Two similar shapes have corresponding sides of 3mm and 9cm. Another side on the larger shape is 4cm. What is the length of the corresponding side on the smaller shape?</p>

Answers: 1) £31.60 2) 1250p 3) 250000 4) £11.25 5) Angles 6) €32 7) 1.5m or 1500m 8) £38.50 9) 9cm 10) £231.48 11) £9 12) 8m 13) 2.5l 14) £312 15) 31.5cm 16) 4.5  
 17) 18.8miles 18) 4.5km 19) £3.75 20) 1.3mm (must have correct units)

# Year 8 - Proportional reasoning...



## Multiplying and Dividing Fractions

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Carry out any multiplication or division using fractions and integers.
- Solutions can be modelled, described and reasoned

### Keywords

**Numerator** : the number above the line on a fraction. The top number. Represents how many parts are taken

**Denominator**: the number below the line on a fraction. The number represent the total number of parts..

**Whole**: a positive number including zero without any decimal or fractional parts.

**Commutative**: an operation is commutative if changing the order does not change the result.

**Unit Fraction**: a fraction where the numerator is one and denominator a positive integer.

**Non-unit Fraction**: a fraction where the numerator is larger than one.

**Dividend** : the amount you want to divide up.

**Divisor**: the number that divides another number.

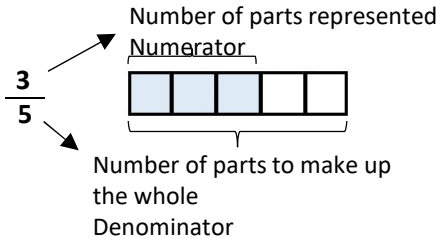
**Quotient**: the answer after we divide one number by another. e.g. dividend ÷ divisor = quotient

**Reciprocal**: a pair of numbers that multiply together to give 1.



### Representing a fraction

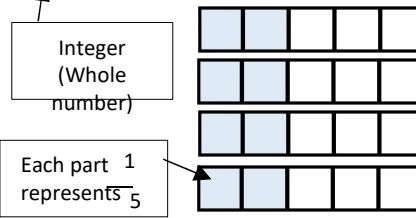
**Numerator**  
**Denominator**



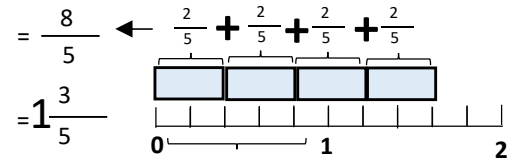
ALL PARTS of a fraction are of equal size

### Repeated addition = multiplication by an integer

$$4 \times \frac{2}{5} \rightarrow \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$$

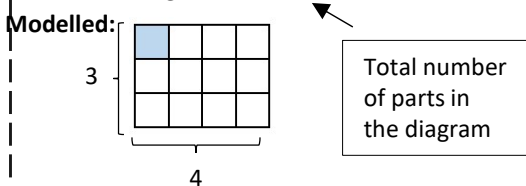


Each whole is split into the same number of parts as the denominator

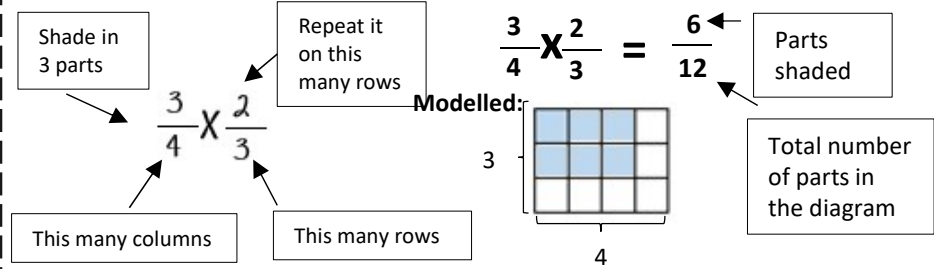


### Multiplying unit fractions

$$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$



### Multiplying non-unit fractions



### Quick Multiplying and Cancelling down

$$\frac{1}{3} \times \frac{4}{9} = \frac{4}{27}$$

The 3 and the 9 have a common factor and can be simplified

**Quick Solving**  
Multiply the numerators  $1 \times 4 = 4$   
Multiply the denominators  $3 \times 9 = 27$

### The reciprocal

When you multiply a number by its reciprocal the answer is always 1

$3 \times \frac{1}{3} = 1$

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

The reciprocal of 3 is  $\frac{1}{3}$  and vice versa

**Reciprocals for division**  
e.g.  $5 \div \frac{1}{4} = 20$

$5 \times 4 = 20$

Multiplying by a reciprocal gives the same outcome

### Dividing an integer by an unit

“There are 4 quarters in 1 whole. Therefore, there are 20 quarters in 5 wholes”

$5 \div \frac{1}{4} = 20$

How many quarters are in 1?

### Dividing any fractions

Remember to use reciprocals

$\frac{2}{5} \div \frac{3}{4}$

$\frac{2}{5} \times \frac{4}{3}$

Multiplying by a reciprocal gives the same outcome

**Represented**

$= \frac{8}{15}$

Multiplying and Dividing Fractions

<p><b>Question 1</b> What is the reciprocal of 5?</p>	<p><b>Question 2</b> Work out <math>\frac{3}{4} \times 3</math></p>	<p><b>Question 3</b> Work out <math>\frac{1}{2} \times \frac{2}{7}</math> SIMPLIFY YOUR ANSWER</p>	<p><b>Question 4</b> Work out <math>\frac{3}{7} \div \frac{1}{3}</math></p>
<p><b>Question 5</b> Work out <math>\frac{5}{7} \times \frac{1}{4}</math></p>	<p><b>Question 6</b> What is the reciprocal of <math>\frac{1}{12}</math>?</p>	<p><b>Question 7</b> Work out <math>\frac{5}{7} \times \frac{1}{3}</math></p>	<p><b>Question 8</b> Work out <math>\frac{4}{11} \div 2</math> SIMPLIFY YOUR ANSWER</p>
<p><b>Question 9</b> Work out <math>\frac{7}{10} \times 5</math></p>	<p><b>Question 10</b> Work out <math>\frac{3}{4} \div \frac{1}{5}</math></p>	<p><b>Question 11</b> What is the reciprocal of <math>\frac{4}{7}</math>?</p>	<p><b>Question 12</b> Work out <math>\frac{1}{2} \div \frac{1}{4}</math></p>
<p><b>Question 13</b> Work out <math>\frac{5}{6} \div \frac{1}{4}</math></p>	<p><b>Question 14</b> Work out <math>\frac{10}{11} \times 5</math></p>	<p><b>Question 15</b> Work out <math>\frac{7}{8} \times \frac{2}{5}</math></p>	<p><b>Question 16</b> What is the reciprocal of -3?</p>
<p><b>Question 17</b> Work out <math>\frac{5}{6} \div \frac{1}{2}</math></p>	<p><b>Question 18</b> What is the reciprocal of 4 as a decimal?</p>	<p><b>Question 19</b> Work out <math>\frac{5}{6} \div \frac{4}{5}</math></p>	<p><b>Question 20</b> Work out <math>\frac{9}{14} \div 6</math></p>

# Year 8 - Proportional reasoning...



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# Ratio and Scale

## What do I need to be able to do?

By the end of this unit you should be able to:

- Simplify any given ratio
- Share an amount in a given ratio
- Solve ratio problems given a part

Solutions should be modelled, explained and solved.

## Keywords

**Ratio:** a statement of how two numbers compare

**Equal Parts::** all parts in the same proportion, or a whole shared equally

**Proportion:** a statement that links two ratios

**Order:** to place a number in a determined sequence

**Part:** a section of a whole

**Equivalent:** of equal value

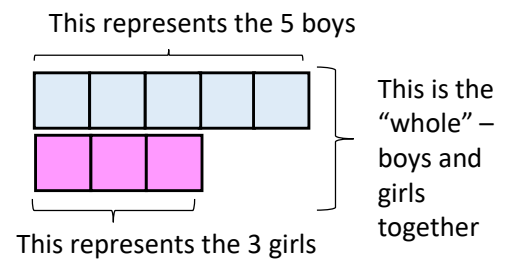
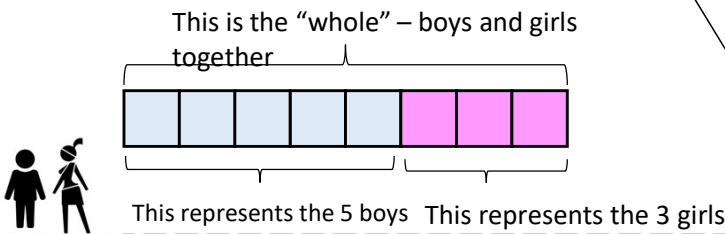
**Factors:** integers that multiply together to get the original value

**Scale:** the comparison of something drawn to its actual size.



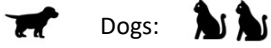
## Representing a ratio

"For every 5 boys there are 3 girls"



## Order is Important Simplifying a ratio

"For every dog there are 2 cats"



Dogs: 1  
Cats: 2

1:2

The ratio has to be written in the same order as the information is given.

e.g. 2:1 would represent 2 dogs for every 1 cat.

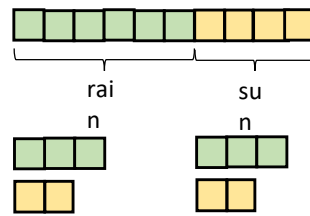
"For every 6 days of rain there are 4 days of sun"

6:4

÷ by 2

3:2

"For every 3 days of rain there are 2 days of sun" – when this happens twice the ratio becomes 6:4.



Cancel down the ratio to its lowest form

Find the biggest common factor that goes into all parts of the ratio

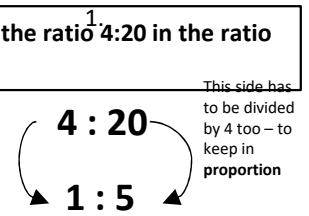
For 6 and 4 the biggest factor (number that multiplies into them is 2)

## Ratio 1:n (or n:1)

This is asking you to cancel down until the part indicated represents

Show the ratio  $1:4:20$  in the ratio of 1:n

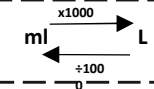
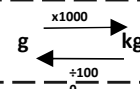
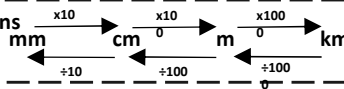
The question states that this part has to be 1 unit. Therefore Divide by 4



## Units are important:

When using a ratio – all parts should be in the same units

Useful Conversions



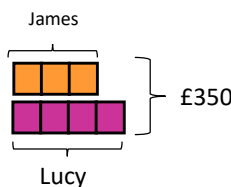
## Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4. Work out how much each person earns

Model the Question

James:  
Lucy

3 : 4



$$£350 \div 7 = £50$$

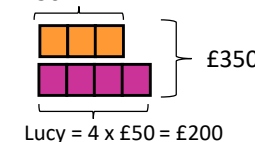
□ = one part = £50

Find the value of one part

Whole: £350  
7 parts to share between (3 James, 4 Lucy)

Put back into the question

James = 3 x £50 = £150



Lucy = 4 x £50 = £200

James: Lucy  
x 50      x 50  
3 : 4  
£150:£200

## Finding a value given 1:n (or n:1)

Inside a box are blue and red pens in the ratio 5:1.

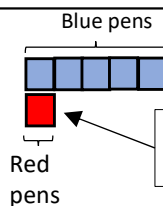
If there are 10 red pens how many blue pens are there?

Model the Question

Blue : Red

5 : 1

□ = one part = 10 pens



Put back into the question

Blue : Red

5 : 1  
x 10      x 10  
50 : 10

Blue pens = 5 x 10 = 50 pens



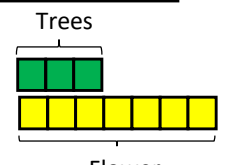
Red pens = 1 x 10 = 10 pens

There are 50 Blue Pens



## Ratio as a fraction

Trees:  
Flowers  
3 : 7



There are 3 parts for trees

Number of parts in group

Total number of parts

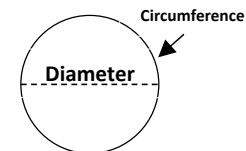
Fraction of trees

3

10

Tree parts 3 + Flower parts 7 = 10

$\frac{\pi}{\pi}$



The ratio of a circles circumference to its diameter

<p><b>Question 1</b> Simplify the ratio 80 : 48</p>	<p><b>Question 2</b> <math>£x</math> is divided in the ratio 5 : 6. The smallest share is £500. What is the difference in the value of the shares?</p>	<p><b>Question 3</b> Divide £300 in the ratio 1 : 5</p>	<p><b>Question 4</b> Express 2 : 220 in the form 1 : <math>n</math></p>	<p><b>Question 5</b> Divide £720 in the ratio 1 : 11</p>
<p><b>Question 6</b> <math>£x</math> is divided in the ratio 3 : 2. The largest share is £90. What is the value of the smallest share?</p>	<p><b>Question 7</b> Express 40 : 136 in the form 1 : <math>n</math></p>	<p><b>Question 8</b> Simplify the ratio 14 : 80</p>	<p><b>Question 9</b> Divide £840 in the ratio 3 : 11</p>	<p><b>Question 10</b> Simplify the ratio 90 : 100</p>
<p><b>Question 11</b> Express 8 : 24 in the form 1 : <math>n</math></p>	<p><b>Question 12</b> <math>£x</math> is divided in the ratio 11 : 10. The largest share is £880. Work out the value of <math>x</math></p>	<p><b>Question 13</b> Simplify the ratio 72 : 54</p>	<p><b>Question 14</b> Express 4 : 24 in the form 1 : <math>n</math></p>	<p><b>Question 15</b> Divide £175 in the ratio 2 : 3</p>
<p><b>Question 16</b> Divide £840 in the ratio 10 : 11</p>	<p><b>Question 17</b> Simplify the ratio 18 : 70</p>	<p><b>Question 18</b> Divide £1235 in the ratio 7 : 6</p>	<p><b>Question 19</b> Express 5 : 50000 in the form 1 : <math>n</math></p>	<p><b>Question 20</b> <math>£x</math> is divided in the ratio 3 : 10. Find the value of <math>x</math> if the difference between the shares is £175</p>

Answers: 1) 5:3 2) £100 3) £50:£250 4) 1:110 5) £60:£660 6) £60 7) 1:3.9 8) 7:40 9) £180:£660 10) 9:10 11) 1:3 12) £1680 13) 4:3 14) 1:6 15) £70:105 16) £400:440 17) 9:35 18) £665:£570 19) 1:10000 20) £325

# Year 8 - Representations...

## Representing Data

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Draw and interpret scatter graphs
- Describe correlation and relationships.
- Identify different types of non-linear relationships.
- Design and complete an ungrouped frequency table.
- Read and interpret grouped tables (discrete and continuous data)
- Represent data in two way tables.

### Keywords

- Variable:** a quantity that may change within the context of the problem.
- Relationship:** the link between two variables (items). E.g. Between sunny days and ice cream sales
- Correlation:** the mathematical definition for the type of relationship..
- Origin:** where two axes meet on a graph.
- Line of best fit:** a straight line on a graph that represents the data on a scatter graph.
- Outlier:** a point that lies outside the trend of graph.
- Quantitative:** numerical data
- Qualitative:** descriptive information, colours, genders, names, emotions etc.
- Continuous:** quantitative data that has an infinite number of possible values within its range.
- Discrete:** quantitative or qualitative data that only takes certain values.
- Frequency:** the number of times a particular data value occurs.

### Draw and interpret a scatter graph.

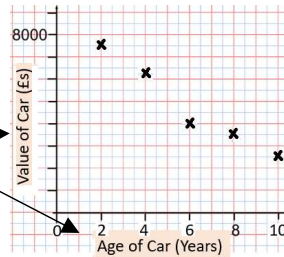
Age of Car (Years)	2	4	6	8	10
Value of Car (Es)	7500	6250	4000	3500	2500

- This data may not be given in size order
- The data forms information pairs for the scatter graph
- Not all data has a relationship

The link between the data can be explained verbally

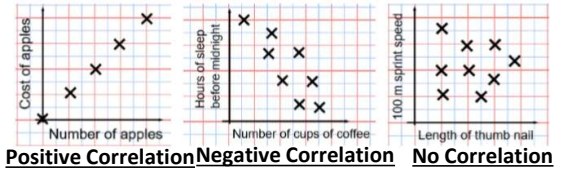
"This scatter graph show as the age of a car increases the value decreases"

All axes should be labelled



The axis should fit all the values on and be equally spread out

### Linear Correlation



**Positive Correlation** **Negative Correlation** **No Correlation**

As one variable increases so does the other variable

As one variable increases the other variable decreases

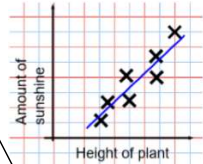
There is no relationship between the two variables

### The line of best fit

The Line of best fit is used to make estimates about the information in your scatter graph

#### Things to know:

- The line of best fit **DOES NOT** need to go through the origin (The point the axes cross)
- There should be approximately the same number of points above and below the line (It may not go through any points)
- The line extends across the whole graph



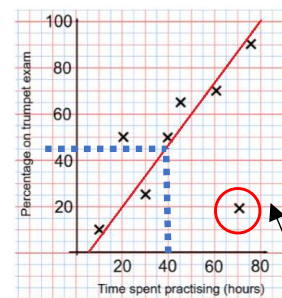
It is only an estimate because the line is designed to be an average representation of the data

It is always a **straight line**.

### Using a line of best fit

**Interpolation** is using the line of best fit to estimate values inside our data point.

e.g. 40 hours revising predicts a percentage of 45.



**Extrapolation** is where we use our line of best fit to predict information outside of our data.

\*\*This is not always useful – in this example you cannot score more than 100%. So revising for longer can not be estimated\*\*

This point is an "outlier" It is an outlier because it doesn't fit this model and stands apart from the data

### Ungrouped Data

The number of times an event happened

The table shows the number of siblings students have. The answers were 3, 1, 2, 2, 0, 3, 4, 1, 1, 2, 0, 2

Number of siblings	Frequency
0	2
1	3
2	4
3	2
4	1

2 people had 0 siblings. This means there are 0 siblings to be counted here

3 + 3 OR 3 x 2 = 6

2 people have 3 siblings so there are 6 siblings in total

**OVERALL there are 0 + 3 + 8 + 6 + 4 Siblings = 21 siblings**

Best represented by discrete data. (Not always a number)

### Grouped Data

If we have a large spread of data it is better to group it. This is so it is easier to look for a trend. Form groups of equal size to make comparison more valid and spread the groups out from the smallest to the largest value.

Cost of TV (£)	Tally	Frequency
101 - 150	7HL II	7
151 - 200	7HL 7HL I	11
201 - 250	7HL	5
251 - 300	III	3

**Discrete Data**  
The groups do not overlap

We do not know the exact value of each item in a group – so an estimate would be used to calculate the overall total (Midpoint)

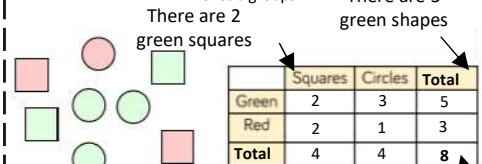
**Continuous Data**  
To make sure all values are included inequalities represent the subgroups

x Weight(g)	Frequency
40 < x ≤ 50	1
50 < x ≤ 60	3
60 < x ≤ 70	5

e.g. this group includes every weight bigger than 60Kg, up to and including 70Kg.

### Representing data in two-way tables

Two-way tables represent discrete information in a visual way that allows you to make conclusions, find probability or find totals of sub groups



#### Using your two-way table

To find a fraction e.g. What fraction of the items are red? **3 red items but 8 items in total =  $\frac{3}{8}$**

**Interleaving:** Use your fraction, decimal percentage equivalence knowledge



### Question 1

What type of correlation describes the link between the amount of rain and the number of bush fires?

### Question 2

Queuing times for lunch is an example of \_\_\_\_\_ data.

Continuous  Discrete

### Question 3

How many people were surveyed when collecting this data?

Number of Pets	Frequency
0	4
1	0
2	3
3	5

### Question 4

Number of TVs	0	1	2	3	4
Frequency	2	5	12	8	3

How many people have one TV?

### Question 5

True or false:

A line of best fit must go through the origin (0,0)

### Question 6

What is wrong with how this grouped frequency table has been constructed?

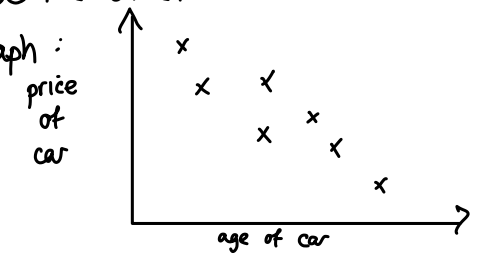
Number of Pets	Frequency
0 - 2	4
2 - 4	0
4 - 6	3
6 - 8	5

### Question 7

What type of correlation describes the link between how far I travel in my car and how much petrol I have?

### Question 8

Describe the correlation shown in the graph:



### Question 9

Number of TVs	0	1	2	3	4
Frequency	2	5	12	8	3

How many people have at least two TVs?

### Question 10

How many pets did the people who were surveyed have?

Number of Pets	Frequency
0	4
1	0
2	3
3	5

### Question 11

The number of people having lunch in the canteen is an example of \_\_\_\_\_ data.

Continuous  Discrete

### Question 12

Marks on a test	Frequency
1 - 10	1
11 - 20	3
21 - 30	6
31 - 40	8

True or false: the range of marks scored on a test is  $40 - 1 = 39$

### Question 13

Complete the two-way table:

	Boys	Girls	Total
Running	6		15
Swimming		12	
Total	19		

### Question 14

What type of correlation describes the link between the time I spend running and the number of calories I burn?

### Question 15

How many students are between 150 and 155cm tall?

Height (x cm)	Frequency
$135 < x \leq 140$	1
$140 < x \leq 145$	3
$145 < x \leq 150$	14
$150 < x \leq 155$	8

### Question 16

True or false:

A line of best fit must be a straight line

### Question 17

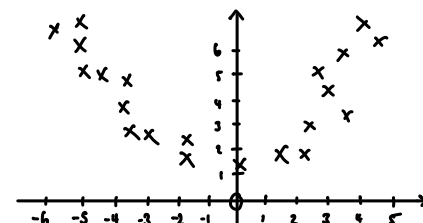
How many students are taller than 145cm?

Height (x cm)	Frequency
$135 < x \leq 140$	1
$140 < x \leq 145$	3
$145 < x \leq 150$	14
$150 < x \leq 155$	8

### Question 18

True or false:

The graph shows no correlation



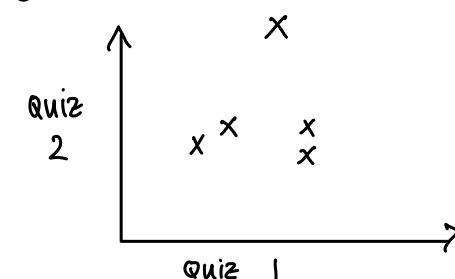
### Question 19

How many TVs are owned altogether?

Number of TVs	0	1	2	3	4
Frequency	2	5	12	8	3

### Question 20

Circle the outlier:



# Year 8 - representations...

# Tables and Probability

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Construct a sample space diagram.
- Systematically list outcomes.
- Find the probability from two-way tables.
- Find the probability from Venn diagrams.

## Keywords

**Outcomes:** the result of an event that depends on probability.

**Probability:** the chance that something will happen.

**Set:** a collection of objects.

**Chance:** the likelihood of a particular outcome.

**Event:** the outcome of a probability – a set of possible outcomes.

**Biased:** a built in error that makes all values wrong by a certain amount.

**Union:** Notation 'U' meaning the set made by comparing the elements of two sets.

## Construct sample space diagrams



Sample space diagrams provide a systematic way to display outcomes from events

The possible outcomes from tossing a

The possible outcomes from rolling a dice

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

This is the set notation to list the outcomes S =

In between the { } are all the possible outcomes

$$S = \{ 1H, 2H, 3H, 4H, 5H, 6H, 1T, 2T, 3T, 4T, 5T, 6T \}$$

## Probability from sample space

The possible outcomes from rolling a dice

The possible outcomes from tossing a

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

This is the set notation that represents the question P

What is the probability that an outcome has an even number and a tails?

There are three even numbers with tails

**Numerator:** the event with tails  
**Denominator:** the total number of outcomes

$$P(\text{Even number and Tails}) = \frac{3}{12}$$

In between the ( ) is the event asked for

There are twelve possible outcomes

## Probability from two-way tables

	Car	Bus	Walk	Total
Boys	15	24	14	53
Girls	6	20	21	47
Total	21	44	35	100

$$P(\text{Girl walk to school}) = \frac{21}{100}$$

The total number of items

The event

The total in the set

## Product Rule

The number of items in event a

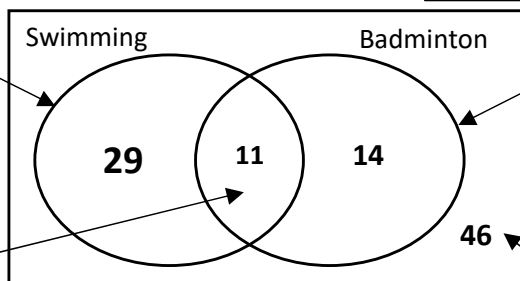
x

The number of items in event b

## Probability from Venn diagrams

This whole curve includes everyone that went swimming. Because 11 did both we calculate just swimming by 40 - 11

The intersection represents both. Swimming AND badminton



100 students were questioned if they played badminton or went to swimming club. 40 went swimming, 25 went to badminton and 11 went to both.

This whole curve includes everyone that went to badminton. Because 11 did both we calculate just badminton by 25 - 11

The number outside represents those that did neither badminton or swimming

$$P(\text{Just swimming}) = \frac{29}{100}$$

$$100 - 29 - 11 - 14$$



**Question 1**

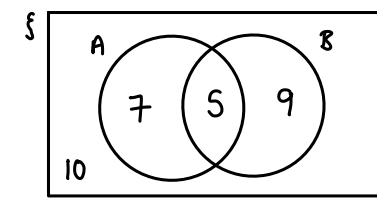
Tommy, Eva and Mo have a race. List all of the possible outcomes for the results.

**Question 2**

	Year 8	Year 9	Total
School Dinner	18	26	39
Packed Lunch	36	25	61
Total	49	51	100

Find the probability of choosing a student who has school dinners.

**Question 3**



Find the probability of picking someone from set A ( $P(A)$ ).

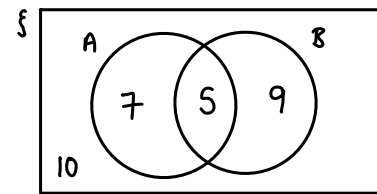
**Question 4**

Rosie has 8 number cards and 2 shape cards and chooses one from each. How many possible outcomes are there?

**Question 5**

Anir has 8 number cards and some shape cards. He chooses one of each card. There are 32 possible outcomes. How many shape cards does he have?

**Question 6**



Find the probability of selecting someone from set A and B. ( $P(A \cap B)$ )

**Question 7**

Complete the sample space for throwing two fair coins:

	Heads	Tails
Heads		
Tails		

**Question 8**

	Year 8	Year 9	Total
School Dinner	18	26	39
Packed Lunch	36	25	61
Total	49	51	100

Find the probability of selecting a Year 9 student who has a packed lunch.

**Question 9**

	Year 8	Year 9	Total
School Dinner	18	26	39
Packed Lunch	36	25	61
Total	49	51	100

Find the probability of selecting a Year 9 student.

**Question 10**

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

Find the probability of getting an even total.

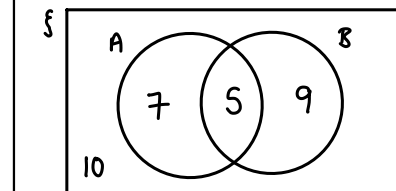
$P(\text{even}) =$

**Question 11**

Two dice are thrown and their scores added together. Complete the sample space:

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

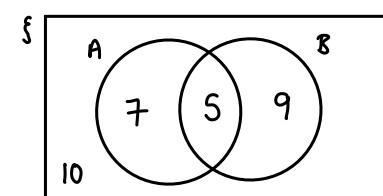
**Question 12**



Find the probability of selecting someone from either set A or B.

$(P(A \cup B))$

**Question 13**



Find the probability of selecting someone not in set A.

$(P(A'))$

**Question 14**

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

Find the probability of selecting a prime number

$P(\text{prime}) =$

**Question 15**

	Year 8	Year 9	Total
School Dinner	18	26	39
Packed Lunch	36	25	61
Total	49	51	100

Find the probability of selecting a Year 9 student who has school dinners.

**Question 16**

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

Find the probability of selecting a square number

$P(\text{square}) =$

**Question 17**

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

Find the probability of selecting an even prime number.

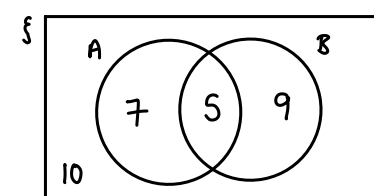
$P(\text{even} \cap \text{prime}) =$

**Question 18**

	Year 8	Year 9	Total
School Dinner	18	26	39
Packed Lunch	36	25	61
Total	49	51	100

Find the probability of selecting a student who has school dinners from the Year 8 students.

**Question 19**



Find the probability of selecting someone who is in set A but is not in set B.

$(P(A \cap B'))$

**Question 20**

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

Find the probability of selecting a number greater than 20

$P(\text{number} > 20) =$

# Year 8 - representations...

## Working in the Cartesian plane

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Label and identify lines parallel to the axes
- Recognise and use basic straight lines
- Identify positive and negative gradients
- Link linear graphs to sequences
- Plot  $y = mx + c$  graphs

### Keywords

**Quadrant:** four quarters of the coordinate plane.

**Coordinate:** a set of values that show an exact position.

**Horizontal:** a straight line from left to right (parallel to the x axis)

**Vertical:** a straight line from top to bottom (parallel to the y axis)

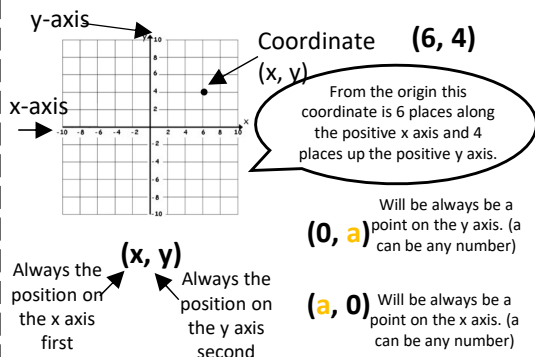
**Origin:** (0,0) on a graph. The point the two axes cross

**Parallel:** Lines that never meet

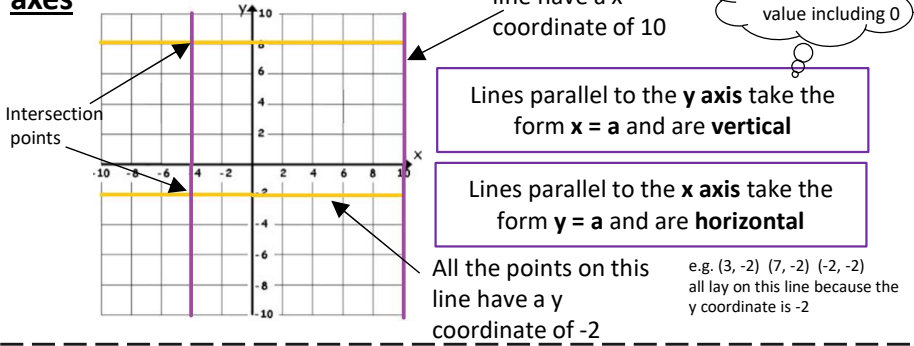
**Gradient:** The steepness of a line

**Intercept:** Where lines cross

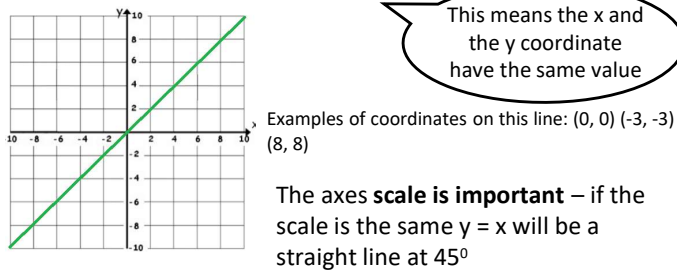
### Coordinates in four quadrants



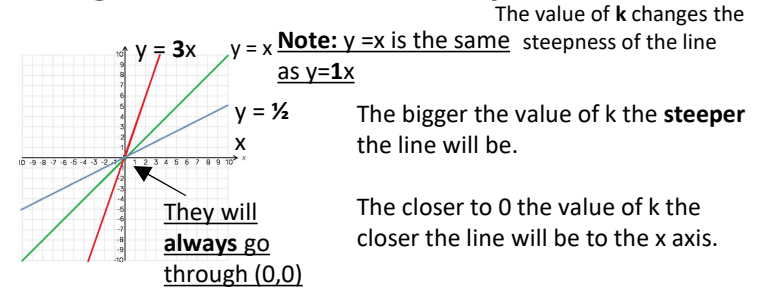
### Lines parallel to the axes



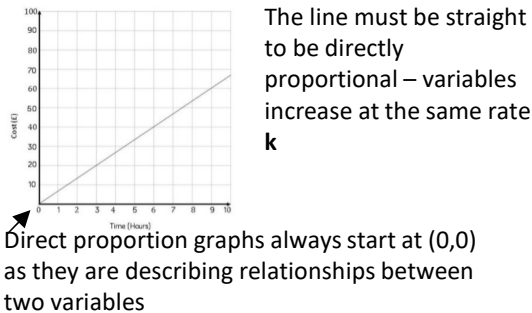
### Recognise and use the line $y=x$



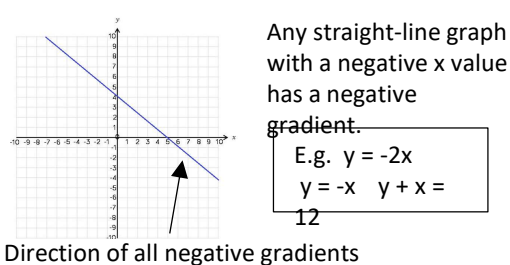
### Recognise and use the lines $y=kx$



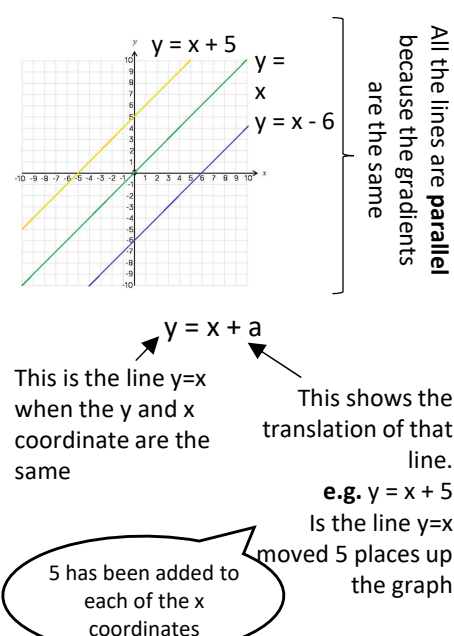
### Direct Proportion using $y=kx$



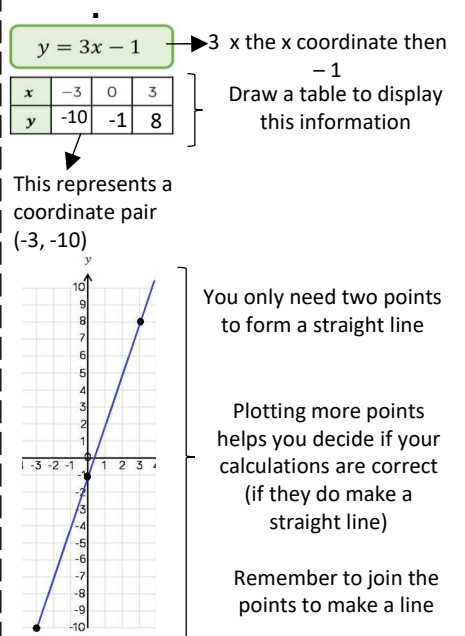
### Lines with negative gradients

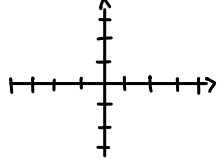
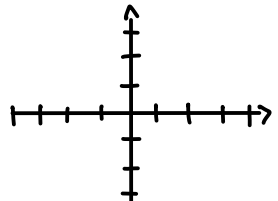
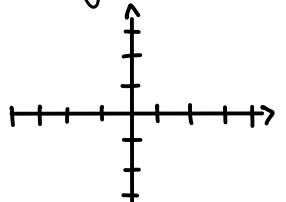
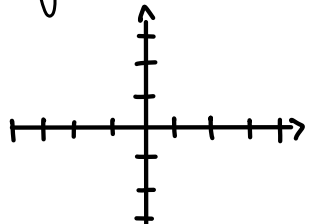
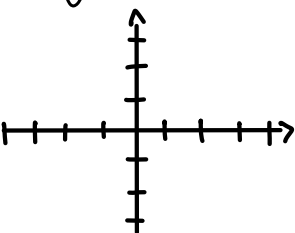
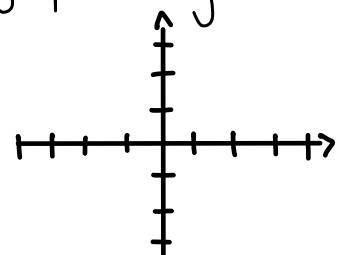
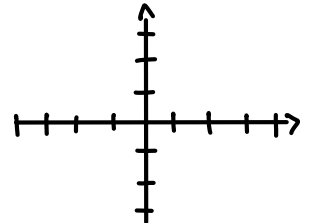


### Lines in the form $y = x + a$



### Plotting $y = mx + c$



<p><b>Question 1</b> Sketch the graph of <math>x=2</math></p> 	<p><b>Question 2</b> Does the point <math>(4, 9)</math> lie on the line <math>y=4x-8</math> ?</p>	<p><b>Question 3</b> The point <math>(0, y)</math> lies on the line <math>y=10x+9</math> Find <math>y</math></p>	<p><b>Question 4</b> Find the midpoint of the line joining points <math>(3, 8)</math> and <math>(8, 6)</math></p>
<p><b>Question 5</b> The point <math>(0, y)</math> lies on the line <math>y=-x-7</math> find <math>y</math></p>	<p><b>Question 6</b> Find the midpoint of the line joining points <math>(-4, -1)</math> and <math>(7, -6)</math></p>	<p><b>Question 7</b> Sketch the graph of <math>x=-1</math></p> 	<p><b>Question 8</b> Does the point <math>(-2, 3)</math> lie on the line <math>y=x+5</math> ?</p>
<p><b>Question 9</b> Sketch the graph of <math>y=3</math></p> 	<p><b>Question 10</b> Does the point <math>(5, -46)</math> lie on the line <math>y=-9x-2</math> ?</p>	<p><b>Question 11</b> Sketch the graph of <math>y=x</math></p> 	<p><b>Question 12</b> The point <math>(0, y)</math> lies on the line <math>y=-x-10</math> Find <math>y</math></p>
<p><b>Question 13</b> The point <math>(1, y)</math> lies on the line <math>y=2x+3</math> Find</p>	<p><b>Question 14</b> Sketch the graph of <math>y=-2</math></p> 	<p><b>Question 15</b> Does the point <math>(6, -20)</math> lie on the line <math>y=-2x-8</math> ?</p>	<p><b>Question 16</b> Find the midpoint of the line joining points <math>(10, -2)</math> and <math>(2, 8)</math></p>
<p><b>Question 17</b> Sketch the graph of <math>y=-x</math></p> 	<p><b>Question 18</b> The point <math>(2, y)</math> lies on the line <math>y=3x-6</math> Find <math>y</math></p>	<p><b>Question 19</b> Does the point <math>(4, 46)</math> lie on the line <math>y=10x+7</math> ?</p>	<p><b>Question 20</b> Sketch the graph of <math>x=0.5</math></p> 

# Year 8 - algebraic techniques...



## Brackets, Equations & Inequalities

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Form Expressions
- Expand and factorise single brackets
- Form and solve equations
- Solve equations with brackets
- Represent inequalities
- Form and solve inequalities

### Keywords

- Simplify:** grouping and combining similar terms  
**Substitute:** replace a variable with a numerical value  
**Equivalent:** something of equal value  
**Coefficient:** a number used to multiply a variable  
**Product:** multiply terms  
**Highest Common Factor (HCF):** the biggest factor (or number that multiplies to give a term)  
**Inequality:** an inequality compares two values showing if one is greater than, less than or equal to another

### Form expressions

For unknown variables, a letter is normally used in its place

- More than – **ADD**  
 Less than/ difference – **SUBTRACT**  
 e.g. 4 more than t →  $t + 4$   
 8 less than k →  $k - 8$

Only similar terms can be grouped together

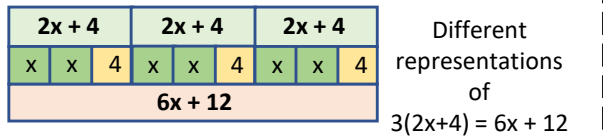
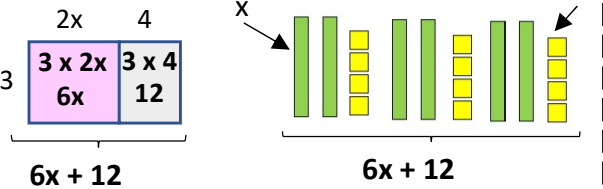
e.g. Find the perimeter of this shape (Perimeter = length around outside of shape)  
 $2t + 1 + t + 2t + 1 + t + 2t + 1 \rightarrow 6t + 2$

### Directed numbers

- $++ \rightarrow +$   
 $-- \rightarrow +$   
 $+- \rightarrow -$   
 $-+ \rightarrow -$

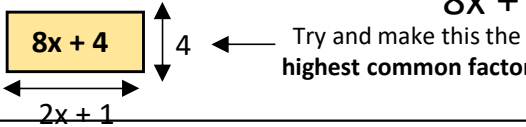
e.g.  $a = -5$  and  $b = 2$   
 $a^2 = a \times a = -5 \times -5 = 25$   
 $b + a = 2 + -5 = -3$

### Multiply single brackets $3(2x + 4)$



### Factorise into a single bracket

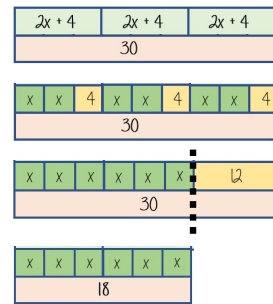
$$8x + 4$$



The two values **multiply** together (also the area) of the rectangle

Note:  
 $8x + 4 \equiv 4(2x + 1)$   
 $8x + 4 \equiv 2(4x + 2)$   
 This is factorised but the HCF has not been used

### Solve equations with brackets $3(2x + 4) = 30$



$$3(2x + 4) = 30$$

Expand the brackets

$$6x + 12 = 30$$

Substitute to check your answer. This could be negative or a fraction or decimal

$$6x = 18$$

$$\div 6 \quad \div 6$$

$$\frac{x}{3} = 3 \quad x = 3$$

### Simple Inequalities

- $<$  less than       $\leq$  Less than or equal to  
 $>$  More than       $\geq$  More than or equal to

$x < 10$   
 Say this out loud "x is a value less than 10"

$10 > x$   
 Say this out loud "10 is more than the value"

Note:  
 $x < 10$  and  $10 > x$  represent the same values

$x + 2 \leq 20$   
 "my value + 2 is less than or equal to 20"  
 $x \leq 18$   
 The biggest the value can be is 18

### Form and solve inequalities



Two more than treble my number is greater than 11

Find the possible range of values

**Form**  $x \rightarrow x3 \rightarrow +2 \rightarrow 11$

$$3x + 2 > 11$$

**Solve**  $x \leftarrow \div 3 \leftarrow -2 \leftarrow 11$

$$x > 3$$

### Check

This would suggest any value bigger than 3 satisfies the statement  
 $3 \times 3 + 2 = 11 \checkmark$        $10 \times 3 + 2 = 32 \checkmark$

### Algebraic constructs

#### Expression

A sentence with a minimum of two numbers and one maths operation

#### Equation

A statement that two things are equal

#### Term

A single number or variable

#### Identity

An equation where both sides have variables that cause the same answer includes  $\equiv$

#### Formula

A rule written with all mathematical symbols

E.g. area of a rectangle  $A = b \times h$

<b>Question 1</b> Simplify $9k - 12k$	<b>Question 2</b> Solve the equation $-5x = 20$	<b>Question 3</b> Expand $3(t+2)$	<b>Question 4</b> Simplify $4d + 3 + -d + 1$
<b>Question 5</b> Write an expression for "two more than $x$ "	<b>Question 6</b> Factorise $3k + 15$	<b>Question 7</b> Expand and Simplify $3(h+5) + 4(h+6)$	<b>Question 8</b> Solve $4(x-7) = 32$
<b>Question 9</b> Expand $4(s+k)$	<b>Question 10</b> List the integers that satisfy the inequality $1 \leq s < 5$	<b>Question 11</b> Solve the inequality $2a + 10 > 4$	<b>Question 12</b> Factorise $49 + 14m$
<b>Question 13</b> Solve $8y + 7 > 3y - 3$	<b>Question 14</b> Expand $2(3p+1)$	<b>Question 15</b> Solve $3(x+5) > 27$	<b>Question 16</b> Expand $x(x-3)$
<b>Question 17</b> Factorise $7m^2 + 14m$	<b>Question 18</b> Expand and Simplify $4(2p+3q) + 2(p+q)$	<b>Question 19</b> Solve $7f + 4 = 4f + 1$	<b>Question 20</b> Expand and Simplify $(x+1)(x+3)$

# Year 8 - algebraic techniques...



## Indices

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Add/ Subtract expressions with indices
- Multiply expressions with indices
- Divide expressions with indices
- Know the addition law for indices
- Know the subtraction law for indices

### Keywords

**Base:** The number that gets multiplied by a power

**Power:** The exponent – or the number that tells you how many times to use the number in multiplication

**Exponent:** The power – or the number that tells you how many times to use the number in multiplication

**Indices:** The power or the exponent.

**Coefficient:** The number used to multiply a variable

**Simplify:** To reduce a power to its lowest term

**Product:** Multiply

### Addition/ Subtraction with indices

$5x^2 + 4x^4$   
 Coefficient      Power  
 $\underbrace{5x^2}_{\text{Term}} + \underbrace{4x^4}_{\text{Term}}$   
 Expression

Each square represents  $x^2$  and each cube represents  $x^4$

Only similar terms can be simplified  
If they have different powers, they are unlike terms

$$5x^2 + 2x^2 \rightarrow \text{7 squares} \rightarrow 7x^2$$

$$5x^2 + 6x^4 - 3x^2 + x^4 \rightarrow \text{2 squares and 7 cubes} \rightarrow 2x^2 + 7x^4$$

### Multiply expressions with indices

$$\begin{aligned} 4b \times 3a &\equiv 4 \times b \times 3 \times a \\ &\equiv 4 \times 3 \times b \times a \\ &\equiv 12 ab \end{aligned}$$

$$\begin{aligned} 5t \times 9t &\equiv 5 \times t \times 9 \times t \\ &\equiv 5 \times 9 \times t \times t \\ &\equiv 45 t^2 \end{aligned}$$

$$\begin{aligned} 2b^4 \times 3b^2 &\equiv 2 \times b \times b \times b \times b \times 3 \times b \times b \\ &\equiv 2 \times 3 \times b \times b \times b \times b \times b \times b \\ &\equiv 6 b^6 \end{aligned}$$

There are often misconceptions with this calculation but break down the powers

### Divide expressions with indices

$$\frac{24}{36} \rightarrow \frac{\cancel{2} \times \cancel{2} \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{3} \times 2 \times \cancel{3}} \rightarrow \frac{2}{3}$$

$$\frac{5a^3 b^2}{15a b^6} \rightarrow \frac{\cancel{5} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{b} \times \cancel{b}}{3 \times \cancel{5} \times \cancel{a} \times \cancel{b} \times \cancel{b} \times \cancel{b} \times \cancel{b} \times \cancel{b} \times \cancel{b}} \rightarrow \frac{a^2}{3b^4}$$

Cross cancelling factors shows cancels the expression

$$\frac{23 a^7 y^2}{5 d b^6}$$

This expression cannot be divided (cancelled down) because there are no common factors or similar terms

### Addition/ Subtraction laws for indices

$$3^5 \times 3^2 \rightarrow 3^7$$

$$= (3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3)$$

The base number is all the same so the terms can be simplified

Addition law for indices

$$a^m \times a^n = a^{m+n}$$

$$3^5 \div 3^2 \rightarrow 3^3$$

$$\frac{3 \times 3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}} \rightarrow \frac{3^3}{3^0} \rightarrow \frac{3^3}{1}$$

Subtraction law for indices

$$a^m \div a^n = a^{m-n}$$

<b>Question 1</b> Simplify $2x^2 + 3x^2$	<b>Question 2</b> Simplify $2x \times 3x$	<b>Question 3</b> Simplify $\frac{15xc}{x}$	<b>Question 4</b> Simplify $x^5 \times x^8$
<b>Question 5</b> Simplify $\frac{3^5}{3^2}$	<b>Question 6</b> Simplify $\frac{15xy}{3y}$	<b>Question 7</b> Simplify $5m^2 + 7m^2 + 2m^2$	<b>Question 8</b> Simplify $4k^2 \times 3k^3$
<b>Question 9</b> Simplify $t \times t \times s \times s \times t$	<b>Question 10</b> Simplify $(a^6)^2$	<b>Question 11</b> Simplify $y^5 \times y^{-2}$	<b>Question 12</b> Simplify $\frac{t^{12}}{t^9}$
<b>Question 13</b> Simplify $(t^{10})^3$	<b>Question 14</b> Simplify $2y^3 + 4y^3 - 5y^3$	<b>Question 15</b> Simplify $y^8 \div y^4$	<b>Question 16</b> Simplify $\frac{15x^2y}{5xy}$
<b>Question 17</b> Simplify $\frac{6ab \times 4ac}{8a}$	<b>Question 18</b> Simplify $x^0$	<b>Question 19</b> Find the value of $x$ $2^7 \times 2^x = 2^{12}$	<b>Question 20</b> Simplify $(5p^4)^2$



# Year 8 - algebraic techniques...



## Sequences

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### What do I need to be able to do?

By the end of this unit you should be able to:

- Generate a sequence from term to term or position to term rules
- Recognise arithmetic sequences and find the nth term
- Recognise geometric sequences and other sequences that arise

### Keywords

**Sequence:** items or numbers put in a pre-decided order

**Term:** a single number or variable

**Position:** the place something is located

**Linear:** the difference between terms increases or decreases (+ or -) by a constant value each time

**Non-linear:** the difference between terms increases or decreases in different amounts, or by x or ÷

**Difference:** the gap between two terms

**Arithmetic:** a sequence where the difference between the terms is constant

**Geometric:** a sequence where each term is found by multiplying the previous one by a fixed non zero number

### Linear and Non Linear Sequences

**Linear Sequences** – increase by addition or subtraction and the same amount each time

**Non-linear Sequences** – do not increase by a constant amount – quadratic, geometric and Fibonacci.

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division.

**Fibonacci Sequence** – look out for this type of sequence

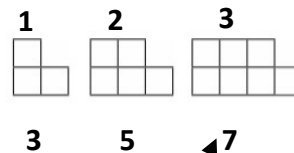
0 1 1 2 3 5 8 ...

Each term is the sum of the previous two terms.



### Sequence in a table and graphically

**Position:** the place in the sequence



"The term in position 3 has 7 squares"

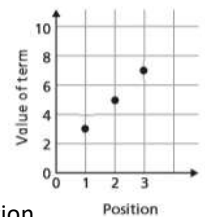
**Term:** the number or variable (the number of squares in each image)

In a table

Position	1	2	3
Term	3	5	7

+2 +2

Graphically



Because the terms increase by the same addition each time this is **linear** – as seen in the graph

### Sequences from algebraic rules

This is substitution!

$$3n + 7$$

This will be linear - note the single power of n. The values increase at a constant rate

$$2n - 5$$

e.g.

$$1^{\text{st}} \text{ term} = 2(1) - 5 = -3$$

$$2^{\text{nd}} \text{ term} = 2(2) - 5 = -1$$

$$100^{\text{th}} \text{ term} = 2(100) - 5 = 195$$

Substitute the number of the term you are looking for in place of 'n'

$$3n^2 + 7$$

This is not linear as there is a power for n

### Checking for a term in a sequence

Form an equation

Is 201 in the sequence  $3n - 4$ ?

$$3n - 4 = 201$$

Term to check

Algebraic rule

Solving this will find the position of the term in the sequence.

ONLY an integer solution can be in the sequence.

### Complex algebraic rules

Misconceptions and comparisons

$$2n^2$$

2 times whatever n squared is

e.g.

$$1^{\text{st}} \text{ term} = 2 \times 1^2 = 2$$

$$2^{\text{st}} \text{ term} = 2 \times 2^2 = 8$$

$$100^{\text{th}} \text{ term} = 2 \times 100^2 = 2000$$

$$(2n)^2$$

2 times n then square the answer

e.g.

$$1^{\text{st}} \text{ term} = (2 \times 1)^2 = 4$$

$$2^{\text{st}} \text{ term} = (2 \times 2)^2 = 16$$

$$100^{\text{th}} \text{ term} = (2 \times 100)^2 = 40000$$

$$n(n + 5)$$

e.g.

$$1^{\text{st}} \text{ term} = 1(1 + 5) = 6$$

$$2^{\text{st}} \text{ term} = 2(2 + 5) = 14$$

$$100^{\text{th}} \text{ term} = 100(100 + 5) = 10500$$

You don't need to expand the expression

### Finding the algebraic rule

This is the 4 times table → 4, 8, 12, 16, 20.....

$$4n$$

7, 11, 15, 19, 22

This has the same constant difference – but is 3 more than the original sequence

$$4n + 3$$

$$4n + 3$$

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence



**Question 1**

Write the first 5 terms of this sequence

"The first term is 3, then add 4 each time"

**Question 2**

List the first 5 square numbers

**Question 3**Find the  $n^{\text{th}}$  term rule:

$$4, 8, 12, 16, \dots$$

**Question 4**

Write the first 5 terms of this sequence described by the rule:

$$4n + 5$$

**Question 5**Find the  $n^{\text{th}}$  term rule:

$$5, 9, 13, 17, \dots$$

**Question 6**

Write the first 5 terms of this sequence described by the rule:

$$10n - 6$$

**Question 7**

Write the first 5 terms of this sequence

"The first term is 10, then subtract 3 each time"

**Question 8**Find the  $n^{\text{th}}$  term rule:

$$-3, 5, 13, 21, \dots$$

**Question 9**

Write the first 5 terms of this sequence described by the rule:

$$4(3n + 1)$$

**Question 10**

Write the first 5 terms of this sequence

"The first term is 1, then double the number each time"

**Question 11**

Write the first 5 terms of this sequence described by the rule:

$$11n - 20$$

**Question 12**

Continue this Fibonacci sequence:

$$1, 1, 2, -, -, -, -, \dots$$

**Question 13**

Describe this sequence using a term-to-term rule:

$$4, 7, 10, 13, 16, \dots$$

**Question 14**Find the  $n^{\text{th}}$  term rule:

$$2, 6, 10, 14, \dots$$

**Question 15**

List the first 5 prime numbers

**Question 16**Find the  $n^{\text{th}}$  term rule:

$$-7, -1, 5, 11, \dots$$

**Question 17**

Write the first 5 terms of this sequence described by the rule:

$$n^3$$

**Question 18**Find the  $n^{\text{th}}$  term rule:

$$\nabla, \nabla \triangle, \nabla \triangle \nabla, \nabla \triangle \triangle, \dots$$

**Question 19**

Write the first 5 terms of this sequence

"The sequence is one less than the 5 times table"

**Question 20**

Does the number 1001 appear in the sequence:

$$-3, 5, 13, 21, \dots \quad ?$$

# Year 8 - Developing number... Fractions & Percentages



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## What do I need to be able to do?

By the end of this unit you should be able to:

- Convert between FDP less than and more than 100.
- Increase or decrease using multipliers.
- Express an amount as a percentage.
- Find percentage change.

## Keywords

- Percent:** parts per 100 – written using the % symbol.  
**Decimal:** a number in our base 10 number system. Numbers to the right of the decimal place are called decimals.  
**Fraction:** a fraction represents how many parts of a whole value you have.  
**Equivalent:** of equal value.  
**Reduce:** to make smaller in value.  
**Growth:** to increase/ to grow.  
**Integer:** whole number, can be positive, negative or zero.  
**Invest:** use money with the goal of it increasing in value over time (usually in a bank).

## Convert FDP

$\frac{70}{100}$  → This also means  $70 \div 100$  → 70 out of 100 squares → 70 "hundredths" = 7 "tenths" = 0.7 → 70 hundredths = 70%

Using a calculator →  $\frac{70}{100}$  → 0.7 → Convert to a decimal

This will give you the answer in the simplest form →  $\frac{70}{100}$  →  $\frac{7}{10}$  → 100 converts to a percentage:

Be careful of recurring decimals  
 e.g.  $\frac{1}{3} = 0.333333$   
 $\frac{2}{3} = 0.\dot{3}$  (The dot above the 3)

## Fraction/ Percentage of amount

Find  $\frac{3}{5}$  of £60 → £60 → £12 £12 £12 £12 £12 → £36

Remember  $\frac{3}{5} = 60\%$

10% of £60 = £6  
 50% of £60 = £30  
 60% of £60 = £36

Remember  $\frac{3}{5} = 60\% = 0.6$   
 60% of £60 =  $0.6 \times 60 = £36$

## Convert FDP < and > 100%

100 hundredths = 10 tenths = 100% → 140 hundredths = 14 tenths = 140% → 40 hundredths = 4 tenths = 40%

$100\% + 40\% = 1 + 0.40 = 1.40$

## Percentage decrease:

**Multipliers** → 100% → 42% → Decrease by 58%

$100\% - 58\% = 42\%$   
 $1.00 - 0.58 = 0.42$

Multiplier Less than 1

## Percentage increase:

**Multipliers** → 100% → 112% → Increase by 12%

$100\% + 12\% = 112\%$   
 $1.00 + 0.12 = 1.12$

Multiplier More than 1

## Express as a % - Non-calculator

7 per every 10 are orange →  $\frac{7}{10}$  → This means that 70 per every 100 are orange →  $\frac{70}{100}$  → 70%

27 per every 50 shaded →  $\frac{27}{50}$  → 54 per every 100 shaded →  $\frac{54}{100}$  → 54%

Denominator 100      Equivalent fractions

## Express as a % - Calculator

Rosie  $\frac{13}{30}$  →  $\frac{13}{30}$  →  $\times 100$  → 43.3333...% → 43%

Can't use equivalence easily to find 'per hundred'

This the same as  $13 \div 30$

Decimal percentages are still a percentage.

## Percentage change

I bought a phone for £200. A year later I sold it for £125.

All values of change compare to the ORIGINAL value

Percentage loss:  $\frac{75}{200} \times \frac{100}{100} = 37.5\%$

I bought a house for £180,000, I later sold it for £216,000.

Percentage profit:  $\frac{36000}{180000} \times 100 = 20\%$

Money made (profit value)

$\frac{\text{Difference in value} \times 100}{\text{Original value}}$

## Choose appropriate method

The language and wording of the question is the key.

Have you represented the question in a bar model?  
 Can you use a calculator?

<p><b>Question 1</b> Express 1 out of 20 as a percentage.</p>	<p><b>Question 2</b> Express 7% as a decimal</p>	<p><b>Question 3</b> What is the multiplier needed for a 25% <u>increase</u>?</p>	<p><b>Question 4</b> Express 2 out of 16 as a percentage.</p>
<p><b>Question 5</b> A car goes down in value from £12,500 to £9,000 in a year. Work out the percentage decrease.</p>	<p><b>Question 6</b> Write in <u>descending</u> order: <math>\frac{19}{50}</math>, 0.4, 39%</p>	<p><b>Question 7</b> Use a calculator to convert <math>\frac{7}{45}</math> to a decimal. Give your answer to 2 decimal places.</p>	<p><b>Question 8</b> What is 0.2 as a percentage?</p>
<p><b>Question 9</b> Find <math>\frac{3}{10}</math> of £280</p>	<p><b>Question 10</b> What is the multiplier needed to work out an increase of 0.6%?</p>	<p><b>Question 11</b> Write in <u>ascending</u> order <math>\frac{4}{25}</math>, 0.15, 17%</p>	<p><b>Question 12</b> Write <math>\frac{5}{100}</math> as a decimal</p>
<p><b>Question 13</b> What is 120% as a fraction?</p>	<p><b>Question 14</b> 20% of <math>x</math> is 20. What is <math>x</math>?</p>	<p><b>Question 15</b> What is 0.45 as a fraction in its simplest form?</p>	<p><b>Question 16</b> The UK population was 67.33 million in 2021. It increased by 0.4% in 2022. What was the population in 2022?</p>
<p><b>Question 17</b> What is <math>\frac{4}{5}</math> of 75?</p>	<p><b>Question 18</b> What is the multiplier needed for a 30% <u>decrease</u>?</p>	<p><b>Question 19</b> Maria spent £35 making a dog kennel and sold it for £55. Work out her percentage profit.</p>	<p><b>Question 20</b> If I use a multiplier of 1.08, will I get an increased amount or a decreased amount?</p>

# Year 8 - Developing number...



# Number Sense

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## What do I need to be able to do?

By the end of this unit you should be able to:

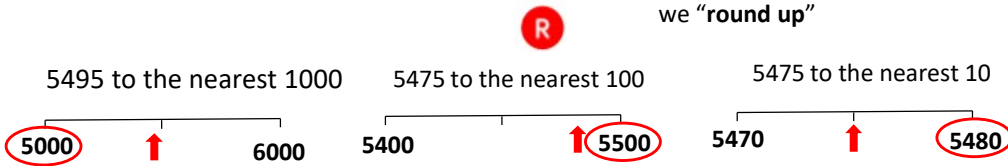
- Round numbers to powers of 10 and 1 sf
- Round numbers to any dp
- Estimate solutions
- Calculate using order of operations
- Calculate with money, units of measurement and time

## Keywords

- Significant:** Place value of importance  
**Round:** Making a number simpler but keeping its value close to what it was.  
**Decimal:** Place holders after the decimal point.  
**Overestimate:** Rounding up – gives a solution higher than the actual value  
**Underestimate:** Rounding down – gives a solution lower than the actual value.  
**Metric:** A system of measurement.  
**Balance:** The amount of money in a bank account  
**Deposit:** Putting money into a bank account.

## Round to powers of 10 and 1 sig. figure

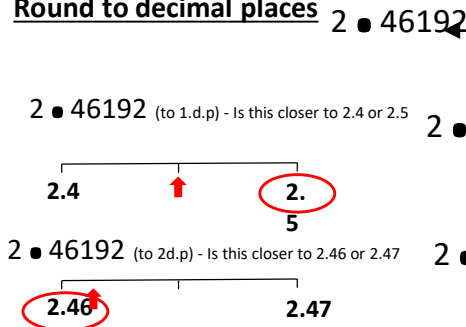
If the number is halfway between we "round up"



- 370 to 1 significant figure is 400
- 37 to 1 significant figure is 40
- 3.7 to 1 significant figure is 4
- 0.37 to 1 significant figure is 0.4
- 0.00037 to 1 significant figure is 0.0004

Round to the first non-zero number

## Round to decimal places



Focus on the numbers after the decimal point  
 This shows the number is closer to 2.5  
 This shows the number is closer to 2.46

## Estimate the calculation

$$4 \bullet 2 + 6 \bullet 7 \approx 4 + 7 \approx 11$$

$$21 \bullet 4 \times 3 \bullet 1 \approx 20 \times 3 \approx 60$$

Round to 1 significant figure to estimate

This is an **overestimate** because the 6.7 was rounded up more

This is an **underestimate** because both values were rounded down

It is good to check all calculations with an estimate in all aspects of maths – it helps you identify calculation errors.

## Order of operations

**Brackets** Operations in brackets are calculated first

**Indices** operations e.g. powers, roots,

**Multiplication/ Division**

They are carried out in the order from left to right in the question

**Addition/ Subtraction**

They are carried out in the order from left to right in the question

## Calculations with money

**Debit** - You have £0 or more in an account

**Credit** - You have less than £0 in an account



Using a calculator – ensure you are working in the correct units.

$$\begin{aligned} \text{£}1.30 + 50\text{p} &= 130 + 50 \text{ (in pence)} \\ &= 1.30 + 0.50 \text{ (in pounds)} \end{aligned}$$

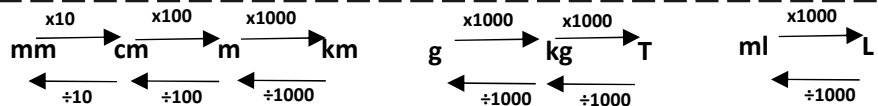
Money calculations are to 2.d.p

$$\text{£}1 = 100\text{p}$$



## Units are important:

Useful Conversions



## Metric measures of length

$$\text{Kilo} = 1000 \times \text{meter} \quad \text{Centi} = \frac{1}{100} \times \text{meter}$$

$$\text{Milli} = \frac{1}{1000} \times \text{meter}$$

## Time and the calendar



**1 Year** – the amount of time it takes Earth to go around the sun 365 (and a quarter) days  
**Leap Year** – 366 days (every 4 years)



**12 Months** = one year = 52 weeks  
 31 days – Jan, March, May, July, Aug, Oct, Dec  
 30 days – April, June, Sept, Nov  
 28 days – Feb (29 leap year)

**1 week** – 7 days  
 Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday

**1 day** – 24 hours  
**1 hour** – 60 minutes  
**1 minute** – 60 seconds

Use a number line for time calculations!

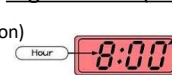
Analogue Clock



12-hour clock

- Use am (morning) and pm (afternoon)
- Only use hour times up to 12

Digital Clock (24-hour times)



Minutes 24-hour clock

- 0-11 (morning hours)
- 12-23 (afternoon hours)

## Units of weight/ capacity

Weight = g, kg, t

Capacity (volume of liquid) = ml, L

<p><b>Question 1</b> Round 372 to the nearest ten.</p>	<p><b>Question 2</b> The minimum recommended sleep time for teenagers is 8 hours. What percentage of a full day is this?</p>	<p><b>Question 3</b> Work out <math>4 + 10 \times 6</math></p>	<p><b>Question 4</b> A coach journey starts at 9:15am and takes <math>2\frac{1}{2}</math> hours. At what time will it arrive?</p>
<p><b>Question 5</b> Convert 0.185 kg to grams.</p>	<p><b>Question 6</b> Round 1,680 to the nearest hundred.</p>	<p><b>Question 7</b> How many cm are in 4m?</p>	<p><b>Question 8</b> Work out <math>33 - 4^2 \times 2</math></p>
<p><b>Question 9</b> <u>Estimate</u> the answer to <math display="block">\frac{21.6 + 18.9}{7.82}</math> (What do you get on a calculator?)</p>	<p><b>Question 10</b> Work out <math>5 \times (21 - 11)</math></p>	<p><b>Question 11</b> Round 32.129 to 1 decimal place.</p>	<p><b>Question 12</b> How many decimal places should we always use when working with money?</p>
<p><b>Question 13</b> Round 0.02845 to 1 significant figure.</p>	<p><b>Question 14</b> Round 32.129 to 2 decimal places.</p>	<p><b>Question 15</b> Nagindra takes his family out for a meal. The bill is £37.85. He pays with a £50 note. What change should he get?</p>	<p><b>Question 16</b> Round 0.368 to the nearest tenth.</p>
<p><b>Question 17</b> Susan's bank account balance is £262.17. She pays a bill and the new balance is £178.30. How much was the bill for?</p>	<p><b>Question 18</b> By rounding the numbers to 1 significant figure, <u>estimate</u> the answer to <math>12.2 \times 4.8</math></p>	<p><b>Question 19</b> Round 4214 to 1 significant figure.</p>	<p><b>Question 20</b> How many days are there in November?</p>

Answers: Q1) 370 Q2) 33.3% Q3) 64 Q4) 11:45am Q5) 185g Q6) 1,700 Q7) 400cm Q8) 1 Q9) 5(5.179...) Q10) 50 Q11) 32.1 Q12) 2 Q13) 0.03 Q14) 32.13 Q15) £12.15 Q16) 0.4 Q17) £83.87 Q18) 40 Q19) 4000 Q20) 30

# Year 8 - Developing number...



# Standard Form

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## What do I need to be able to do?

By the end of this unit you should be able to:

- Write numbers in standard form and as ordinary numbers
- Order numbers in standard form
- Add/ Subtract with standard form
- Multiply/ Divide with standard form
- Use a calculator with standard form

## Keywords

**Standard (index) Form:** A system of writing very big or very small numbers

**Commutative:** an operation is commutative if changing the order does not change the result.

**Base:** The number that gets multiplied by a power

**Power:** The exponent – or the number that tells you how many times to use the number in multiplication

**Exponent:** The power – or the number that tells you how many times to use the number in multiplication

**Indices:** The power or the exponent.

**Negative:** A value below zero.

## Positive powers of 10

1 billion – 1 000 000 000

$$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^9$$

$$\text{Addition rule for indices } 10^a \times 10^b = 10^{a+b}$$

$$\text{Subtraction rule for indices } 10^a \div 10^b = 10^{a-b}$$

## Numbers between 0 and 1

0 ● 05.4	1	●	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
= 5 ● 4 x 10 <sup>-2</sup>	10 <sup>0</sup>	●	10 <sup>-1</sup>	10 <sup>-2</sup>	10 <sup>-3</sup>
	0	●	0	5	4

A negative power does not mean a negative answer – it means a number closer to 0

## Standard form with numbers > 1

Any number between 1 and less than 10  $A \times 10^n$  ← Any integer

### Example

$$3.2 \times 10^4 = 3.2 \times 10 \times 10 \times 10 \times 10 = 32000$$

### Non-example

$$0.8 \times 10^4 = 8000$$

$$5.3 \times 10^{0.7}$$

## Negative powers of 10

0.001	10	1	●	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
$1 \times \frac{1}{1000}$	10 <sup>1</sup>	10 <sup>0</sup>	●	10 <sup>-1</sup>	10 <sup>-2</sup>	10 <sup>-3</sup>
$1 \times 10^{-3}$	0	0	●	0	0	1

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

## Order numbers in standard form

6 ● 4 x 10 <sup>-2</sup>	2 ● 4 x 10 <sup>2</sup>	3 ● 3 x 10 <sup>0</sup>	1 ● 3 x 10 <sup>-1</sup>	10 <sup>2</sup>	10 <sup>1</sup>	10 <sup>0</sup>	●	10 <sup>-1</sup>	10 <sup>-2</sup>	10 <sup>-3</sup>	10 <sup>-4</sup>
0 ● 064	240	1	0 ● 13	Look at the power first will the number be = > or < than 1							
Use a place value grid to compare the numbers for ordering											

## Mental calculations

6 ● 4 x 10<sup>2</sup> x 1000 Not in Standard Form (8) x 10<sup>5</sup> x (3)

$$= 6 \bullet 4 \times 10^2 \times 10^3 = 24 \times 10^5 \text{ Not in Standard Form}$$

$$= 2 \bullet 4 \times 10^1 \times 10^5 = 2 \bullet 4 \times 10^6$$

Use addition for indices rule

Use addition for indices rule

$$(2 \times 10^3) \div 4 = (2 \div 4) \times 10^3 \text{ Divide the values} = 0 \bullet 5 \times 10^3$$

## Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard form at the end.

Method 1

$$6 \times 10^5 + 8 \times 10^5 = 600000 + 800000 = 1400000 = 1 \bullet 4 \times 10^6$$

Method 2

$$= (6 + 8) \times 10^5 = 14 \times 10^5 = 1 \bullet 4 \times 10^1 \times 10^5 = 1 \bullet 4 \times 10^6$$

More robust method  
Less room for misconceptions  
Easier to do calculations with negative indices  
Can use for different powers

This is not the final answer

Only works if the powers are the same

## Multiplication and division

1.5 x 10<sup>5</sup> / 0.3 x 10<sup>3</sup> Division questions can look like this

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations

$$(1.5 \times 10^5) \div (0.3 \times 10^3) = 1.5 \div 0.3 \times 10^5 \div 10^3 = 5 \times 10^2$$

Revisit addition and subtraction laws for indices – they are needed for the calculations

Addition law for indices  $a^m \times a^n = a^{m+n}$

Subtraction law for indices  $a^m \div a^n = a^{m-n}$

## Using a calculator

1.4 x 10<sup>5</sup> x 3.9 x 10<sup>3</sup>

Input 1.4 and pres  $\times 10^x$  hen press 5 (for the power)  
Press  $\times$

Input 3.9 and pres:  $\times 10^x$  Then press 3 (for the power)  
Press  $=$

This gives you the solution

Answer: 5.5 x 10<sup>8</sup>

Use a calculator to work out this question to a suitable degree of accuracy.



<b>Question 1</b> Convert $2.5 \times 10^4$ to an ordinary number.	<b>Question 2</b> Convert 478,250 into Standard Form.	<b>Question 3</b> Which of these is in correct standard form? A. $6 \times 6^2$ B. $3 \times 10^4$ C. $58 \times 10^6$	<b>Question 4</b> Calculate $2.6 \times 10^{-3} \times 6.8 \times 10^{-4}$ Give your answer in Standard Form
<b>Question 5</b> Work out $7 \times 10^3 \times 8 \times 10^2$ Give your answer in standard form.	<b>Question 6</b> What is the value of $10^3$ ?	<b>Question 7</b> Convert 74,800 to standard form.	<b>Question 8</b> Using the addition rule for indices, what is $10^6 \times 10^2$ ?
<b>Question 9</b> Work out $2.5 \times 10^3 \div 5 \times 10^6$ Give your answer in Standard Form.	<b>Question 10</b> Which one of these is in correct standard form? A: $32 \times 10^{14}$ B: $8.9 \times 10^{-3}$ C: $6.2 \times 4^{10}$	<b>Question 11</b> What is $10^{-2}$ as an ordinary number?	<b>Question 12</b> Work out $7.5 \times 10^5 - 4.2 \times 10^3$ Give your answer as an ordinary number.
<b>Question 13</b> Calculate $3 \times 10^6 + 4.2 \times 10^7$ Give your answer in Standard form.	<b>Question 14</b> Work out $3 \times 10^2 + 5 \times 10^3$ Give your answer in Standard form.	<b>Question 15</b> Which one of these numbers is a good estimate for the diameter of the earth? A: $2.78 \times 10^{-11}$ km B: $1.27 \times 10^5$ km	<b>Question 16</b> Adjust $23 \times 10^4$ so that it is in correct standard form.
<b>Question 17</b> One atom of iron (Fe) has a radius of $1.26 \times 10^{-10}$ m. Write this as an ordinary number.	<b>Question 18</b> Use the subtraction rule for indices to work out $\frac{10^6}{10^4} = \square$	<b>Question 19</b> Work out $\frac{2.7 \times 10^4}{3 \times 10^2}$ Give your answer in standard form.	<b>Question 20</b> What is 1 million written as a power of 10?

ANSWERS: Q1) 25,000 Q2)  $4.7825 \times 10^5$  Q3) B Q4)  $1.768 \times 10^{-6}$  Q5)  $5.6 \times 10^6$  Q6) 1000 Q7)  $7.48 \times 10^4$  Q8)  $10^8$  Q9)  $5 \times 10^{-4}$  Q10) B Q11) 0.01 or  $\frac{1}{100}$  Q12) 745800 Q13)  $4.5 \times 10^7$  Q14)  $5.3 \times 10^3$  Q15) B Q16)  $2.3 \times 10^5$  Q17) 0.0000000126 (9 zeroes) Q18)  $10^2$  Q19)  $9 \times 10^1$  Q20)  $10^6$



# Year 8 - Developing geometry...

## Angles in parallel lines and polygons



@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Identify alternate angles
- Identify corresponding angles
- Identify co-interior angles
- Find the sum of interior angles in polygons
- Find the sum of exterior angles in polygons
- Find interior angles in regular polygons

### Keywords

**Parallel:** Straight lines that never meet

**Angle:** The figure formed by two straight lines meeting (measured in degrees)

**Transversal:** A line that cuts across two or more other (normally parallel) lines

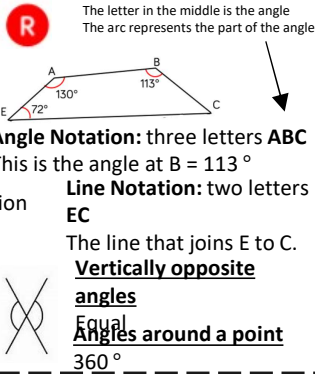
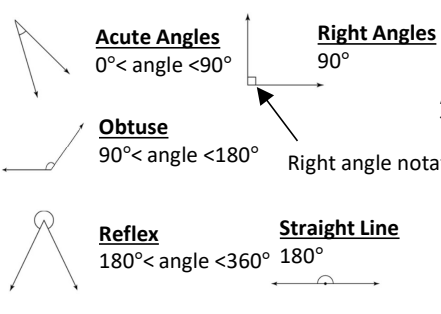
**Isosceles:** Two equal size lines and equal size angles (in a triangle or trapezium)

**Polygon:** A 2D shape made with straight lines

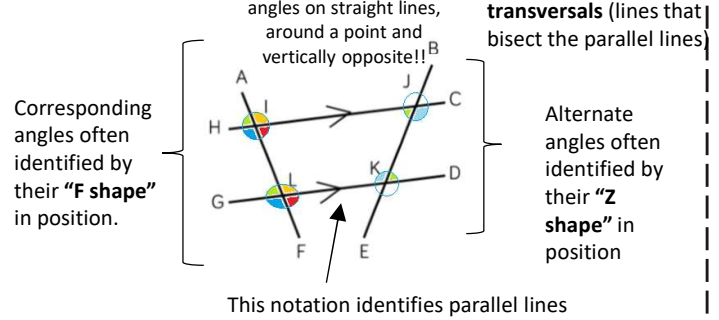
**Sum:** Addition (total of all the interior angles added together)

**Regular polygon:** All the sides have equal length; all the interior angles have equal size.

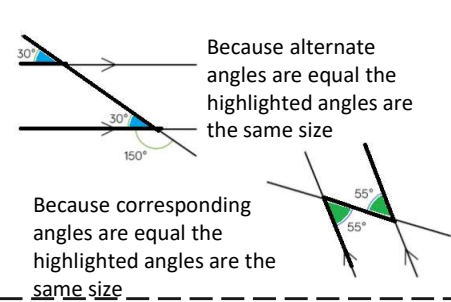
### Basic angle rules and notation



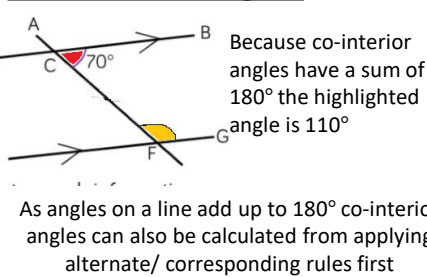
### Parallel lines



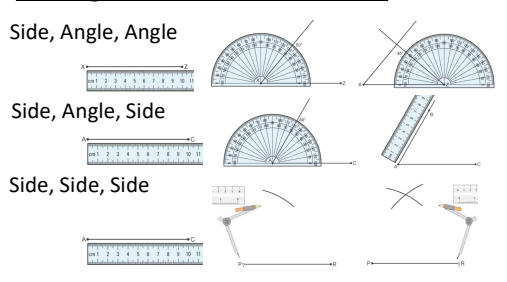
### Alternate/ Corresponding angles



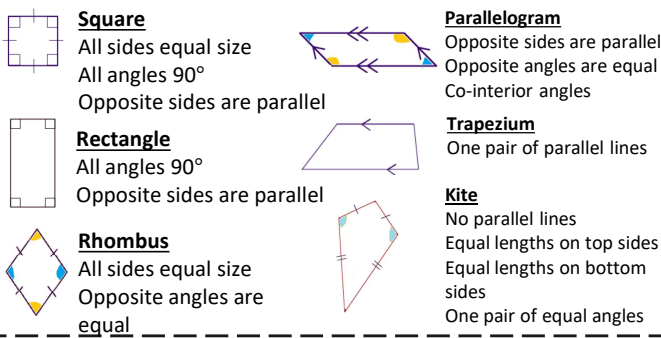
### Co-interior angles



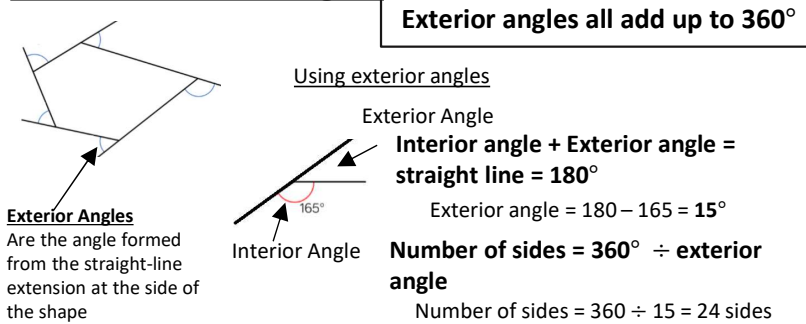
### Triangles & Quadrilaterals



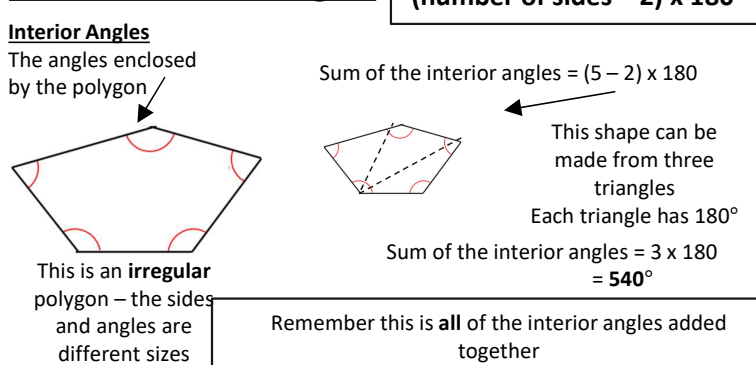
### Properties of Quadrilaterals



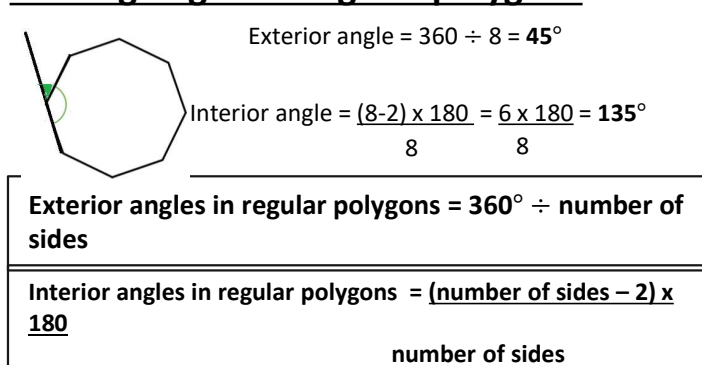
### Sum of exterior angles



### Sum of interior angles



### Missing angles in regular polygons



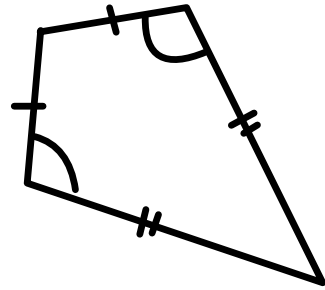
**Question 1**  
An angle of  $28^\circ$  is:  
A) acute B) reflex C) obtuse

**Question 2**  
Complete the sentence:  
Alternate angles in parallel lines are \_\_\_\_.

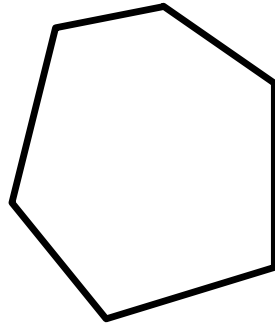
**Question 3**  
What is the sum of exterior angles of a 12 sided polygon?

**Question 4**  
What is the sum of the interior angles of a nonagon?

**Question 5**  
What is the name of this quadrilateral?



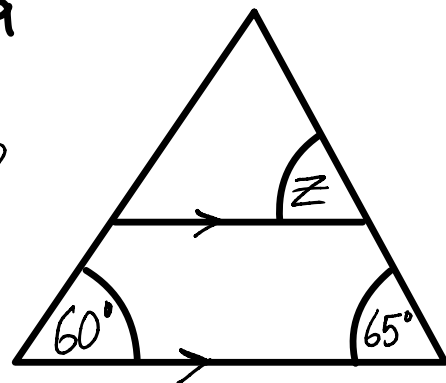
**Question 6**  
How many triangles can I draw inside a hexagon starting from the same vertex?



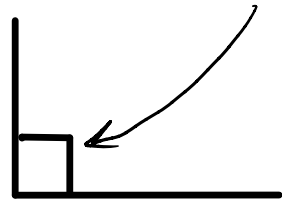
**Question 7**  
What is the formula to calculate the sum of interior angles of any polygon?

**Question 8**  
Complete the sentence:  
Corresponding angles in parallel lines are \_\_\_\_.

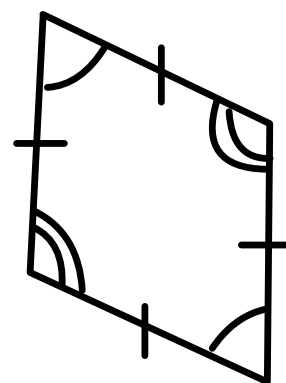
**Question 9**  
What is angle  $z$ ?



**Question 10**  
What does this symbol mean?



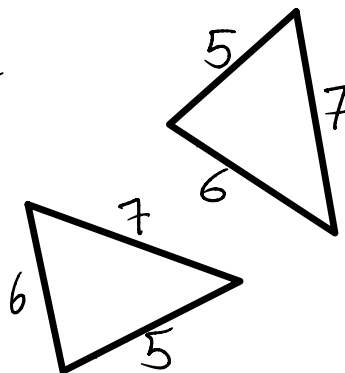
**Question 11**  
What is the name of this quadrilateral?



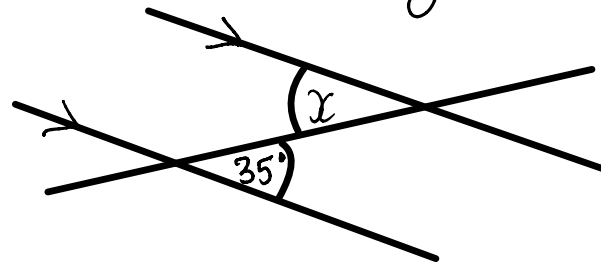
**Question 12**  
How many sides does a regular polygon have if one of its exterior angles is  $18^\circ$ ?

**Question 13**  
Complete the sentence:  
Co-interior angles in parallel lines \_\_\_\_.

**Question 14**  
Which rule says that these two triangles are congruent? (the same)

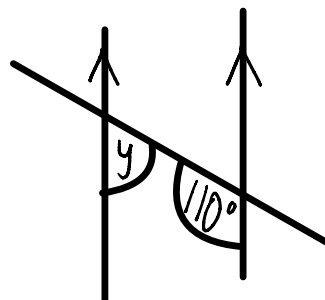


**Question 15**  
What size is angle  $x$ ?

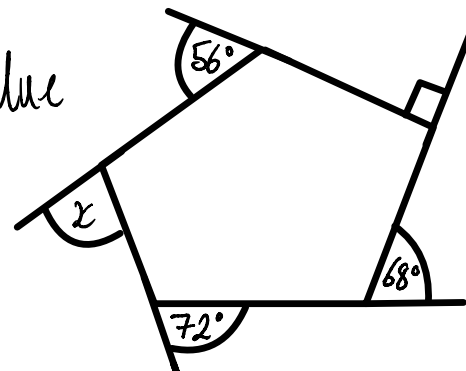


**Question 16**  
The exterior angles of any polygon always add to \_\_\_\_.

**Question 17**  
What size is angle  $y$ ?



**Question 18**  
Find the value of  $x$



**Question 19**  
What is special about a "regular" polygon?

**Question 20**  
Calculate the size of one interior angle of a regular octagon.

(19) Sides and angles are the same

(20)  $135^\circ$

Answers: (1) Acute (2) equal (3)  $360^\circ$  (4)  $1260^\circ$  (5) Kite (6) 4 (7)  $(n-2) \times 180^\circ$  (8) equal (9)  $65^\circ$  (10) Right Angle (11) Rhombus (12) 20 (13) add to  $180^\circ$  (14) SSS (15)  $35^\circ$  (16)  $360^\circ$  (17)  $70^\circ$  (18)  $74^\circ$

# Year 8 - Developing geometry...

## Area of trapezia and Circles

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Recall area of basic 2D shapes
- Find the area of a trapezium
- Find the area of a circle
- Find the area of compound shapes
- Find the perimeter of compound shapes

### Keywords

**Congruent:** The same

**Area:** Space inside a 2D object

**Perimeter:** Length around the outside of a 2D object

**Pi ( $\pi$ ):** The ratio of a circle's circumference to its diameter.

**Perpendicular:** At an angle of  $90^\circ$  to a given surface

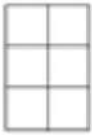
**Formula:** A mathematical relationship/ rule given in symbols. E.g.  $b \times h =$  area of rectangle/ square

**Infinity ( $\infty$ ):** A number without a given ending (too great to count to the end of the number) – never ends

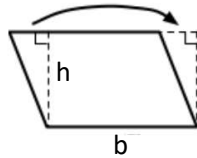
**Sector:** A part of the circle enclosed by two radii and an arc.

### Area – rectangles, triangles, parallelograms

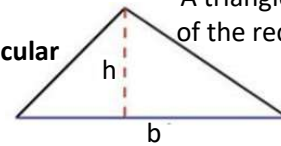
Rectangle  
Base x Height



Parallelogram/  
Rhombus  
Base x Perpendicular height



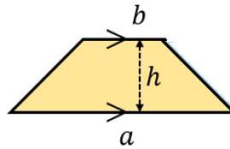
Triangle  
 $\frac{1}{2} \times$  Base x Perpendicular height



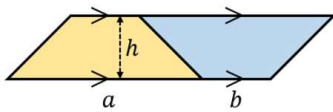
A triangle is half the size of the rectangle it would fit in

### Area of a trapezium

Area of a trapezium  
 $\frac{(a + b) \times h}{2}$



Why?

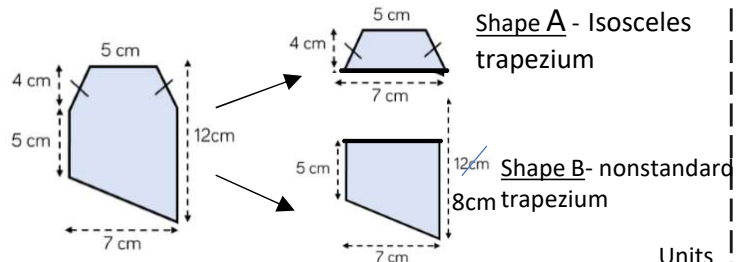


- Two congruent trapeziums make a parallelogram
- New length  $(a + b) \times$  height
- Divide by 2 to find area of one

### Compound shapes

To find the area compound shapes often need splitting into more manageable shapes first.

Identify the shapes and missing sides etc. first.



Shape A - Isosceles trapezium

Shape B - nonstandard 8cm trapezium

Shape A + Shape B = total area

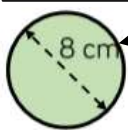
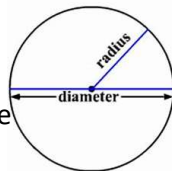
$$\frac{(5+7) \times 4}{2} + \frac{(5+8) \times 7}{2} = 24 + 45 = 69 \text{ cm}^2$$

Units

### Area of a circle (Non-Calculator)

Read the question – leave in terms of  $\pi$  or if  $\pi \approx 3$  (provides an estimate for answers)

Area of a circle  
 $\pi \times \text{radius}^2$



Diameter = 8cm  
 $\therefore$  Radius = 4cm

Radius = 4cm

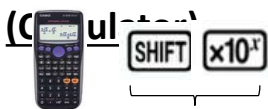
$$\begin{aligned} \pi \times \text{radius}^2 \\ = \pi \times 4^2 \\ = \pi \times 16 \\ = 16\pi \text{ cm}^2 \end{aligned}$$

Find the area of one quarter of the circle

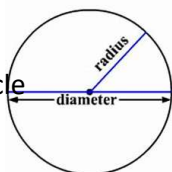


Circle Area =  $16\pi \text{ cm}^2$   
Quarter =  $4\pi \text{ cm}^2$

### Area of a circle



Area of a circle  
 $\pi \times \text{radius}^2$



How to get  $\pi$  symbol on the calculator

It is important to round your answer suitably – to significant figures or decimal places. This will give you a decimal solution that will go on forever!

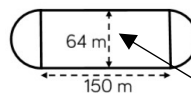
### Compound shapes including circles

Circumference  
 $\pi \times \text{diameter}$

Compound shapes are not always area questions.

For Perimeter you will need to use the circumference

Spotting diameters and radii



This dimension is also the diameter of the semi circles.

$$\begin{aligned} \text{Arc lengths} &= \pi \times 64 \\ &= 64\pi \end{aligned}$$

Don't need to halve this because there are 2 ends which make the whole circle

Arc lengths + Straight lengths = total perimeter

$$\begin{aligned} &= 64\pi + 150 + 150 \\ &= (300 + 64\pi) \text{ m} \\ \text{OR} &= \underline{501.1 \text{ m}} \end{aligned}$$

Still remember to split up the compound shape into smaller more manageable individual shapes first

**Question 1**

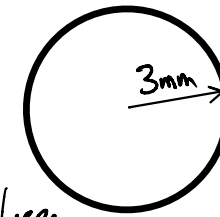
Look at the drawing of a trapezium on the K.O. Now complete the sentence: "A trapezium is a quadrilateral with one pair of \_\_\_\_\_ sides."

**Question 2**

What should you use as an approximation of  $\pi$  if you are working out an estimate of the area of a circle?

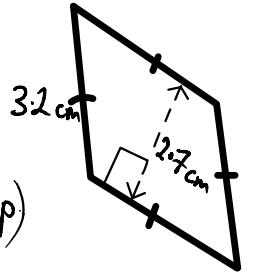
**Question 3**

Calculate the area of this circle. Give your answer to 2 decimal places.



**Question 4**

Work out the area of this rhombus. (Give your answer to 1 d.p.)



**Question 5**

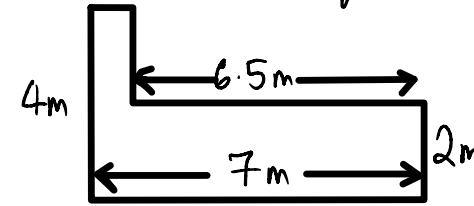
What is the formula for the area of a circle?

**Question 6**

A circle has a diameter of 14cm. What is its radius?

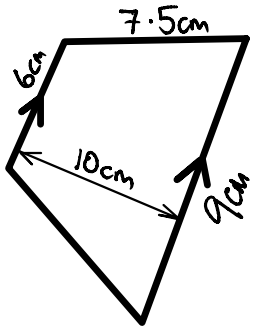
**Question 7**

Work out the area of this compound shape.



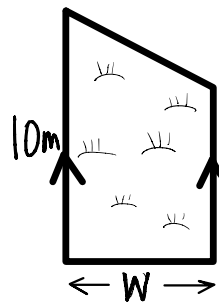
**Question 8**

Calculate the area of this trapezium. (Don't forget units)



**Question 9**

If the area of this lawn is  $48m^2$ , how wide must it be?

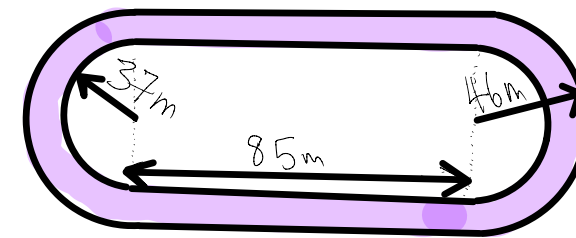


**Question 10**

A circular pond has an area of  $78m^2$ . How wide is the pond? Give your answer to the nearest integer.

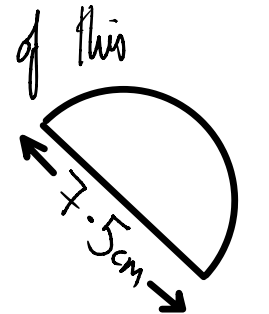
**Question 11**

What is the shaded area of this running track?



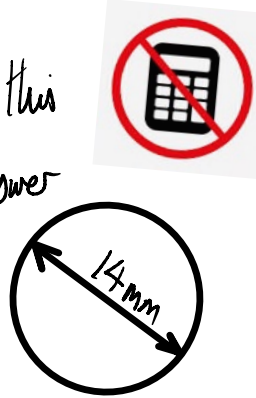
**Question 12**

What is the area of this semi-circle? (Round your answer to 3 sig figs)



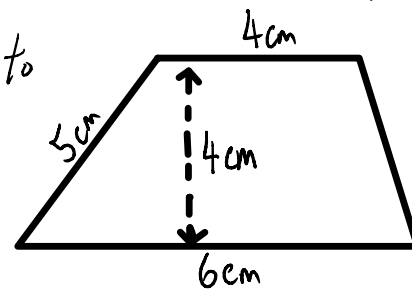
**Question 13**

Calculate the area of this circle. Leave your answer in terms of  $\pi$ . (Don't forget units).



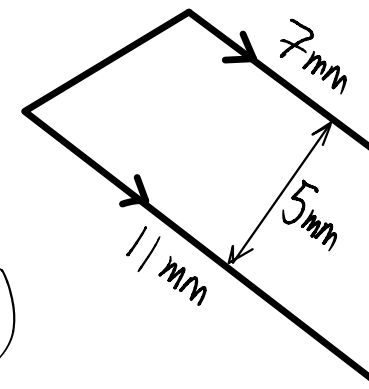
**Question 14**

Which of the dimensions on this trapezium is not needed to calculate its area?



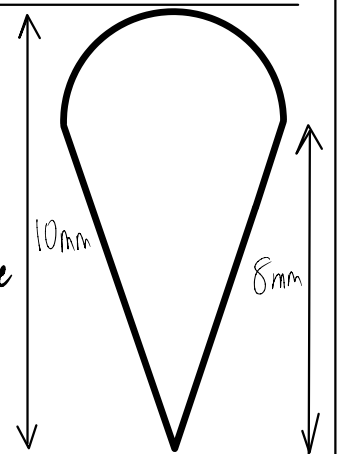
**Question 15**

Use the formula to work out the area of this trapezium. (Include correct units)



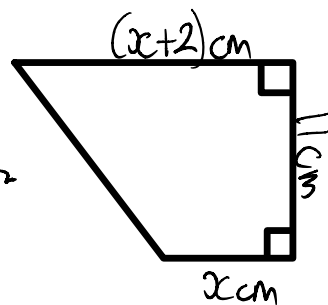
**Question 16**

Work out the area of this logo outline made from a triangle and a semi-circle.



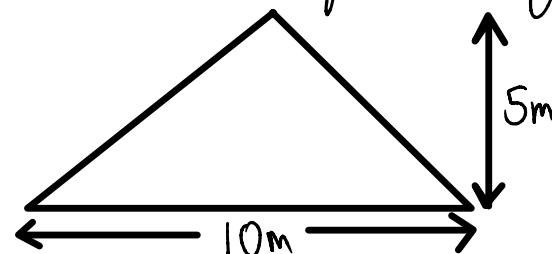
**Question 17**

The area of this trapezium is  $55cm^2$ . Set up an equation and solve it to find  $x$ .



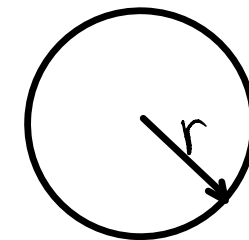
**Question 18**

What is the area of this triangle?



**Question 19**

This circle has an area of  $78cm^2$ . Find the radius,  $r$ . (Round to 2 sig. fig.)



**Question 20**

What is the length of the inside lane of the running track in Question 11? (Give your answer to nearest 10m)



# Year 8 - Developing geometry... Line symmetry and reflection



@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Recognise line symmetry
- Reflect in a horizontal line
- Reflect in a vertical line
- Reflect in a diagonal line

## Keywords

- Mirror line:** a line that passes through the center of a shape with a mirror image on either side of the line
- Line of symmetry:** same definition as the mirror line
- Reflect:** mapping of one object from one position to another of equal distance from a given line.
- Vertex:** a point where two or more-line segments meet.
- Perpendicular:** lines that cross at  $90^\circ$
- Horizontal:** a straight line from left to right (parallel to the x axis)
- Vertical:** a straight line from top to bottom (parallel to the y axis)

## Lines of symmetry

Mirror line (line of reflection)

Rhombus  
two lines of symmetry

Parallelogram  
No lines of symmetry

Shapes can have more than one line of symmetry....  
This regular polygon (a regular pentagon has 5 lines of symmetry)

A circle has an infinite amount of lines of symmetry

## Reflect horizontally/ vertically (1)

Reflection in a vertical line

Reflection in a horizontal line

Note: a reflection doubles the area of the original shape

Reflection on an axis grid

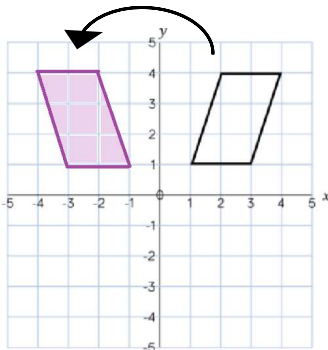
Reflection in the line  $y=2$

Reflection in the line  $x=2$

## Reflect horizontally/ vertically (2)

All points need to be the same distance away from the line of reflection

Reflection in the line  $y$  axis – this is also a reflection in the line  $x=0$



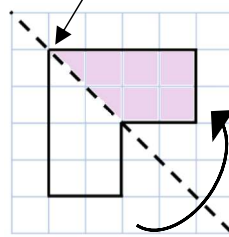
### Lines parallel to the x and y axis

REMEMBER

- Lines parallel to the x-axis are  $y = \underline{\hspace{2cm}}$
- Lines parallel to the y-axis are  $x = \underline{\hspace{2cm}}$

## Reflect Diagonally (1)

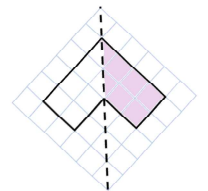
Points on the mirror line don't change position



Fold along the line of symmetry to check the direction of the reflection

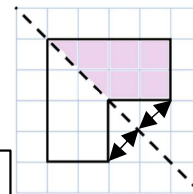
### Turn your image

If you turn your image it becomes a vertical/horizontal reflection (also good to check your answer this way)



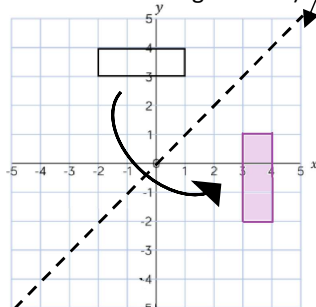
### Drawing perpendicular lines

Perpendicular lines to and from the mirror line can help you to plot diagonal reflections

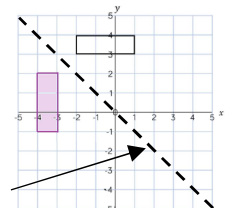


## Reflect Diagonally (2)

This is the line  $y = x$  (every  $y$  coordinate is the same as the  $x$  coordinate along this line)

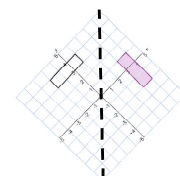


This is the line  $y = -x$   
The  $x$  and  $y$  coordinate have the same value but opposite sign



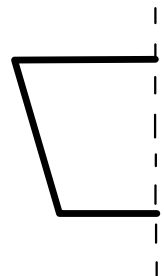
### Turn your image

If you turn your image it becomes a vertical/horizontal reflection (also good to check your answer this way)



**Question 1**

Reflect this shape in the mirror line



**Question 2**

Straight lines parallel to the x-axis have equations of the form:

- A)  $x = (\text{a constant})$  B)  $y = (\text{a constant})$

**Question 3**

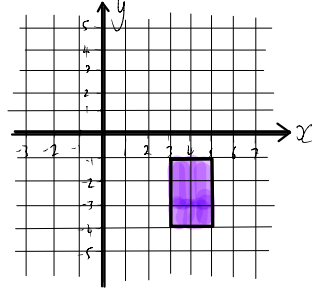
How many lines of symmetry does an equilateral triangle have?

**Question 4**

The line  $x = 0$  is also called the \_\_\_-axis

**Question 5**

Reflect this shape in the line  $y = 0$

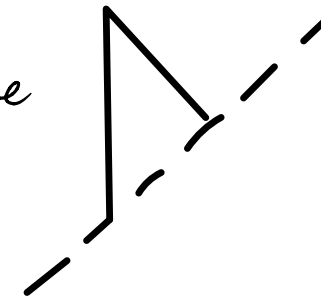


**Question 6**

Complete the sentence: All points need to be the \_\_\_\_\_ away from the line of \_\_\_\_\_.

**Question 7**

Reflect the shape in the mirror line



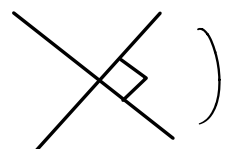
**Question 8**

"A hexagon has 6 lines of symmetry." Is this true: Always, Sometimes or Never?

**Question 9**

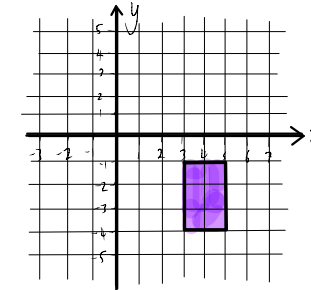
For Question 8, when is it certain to say that a hexagon has 6 lines of symmetry?

**Question 10**

The lines  $y = x$  and  $y = -x$  are \_\_\_\_\_ to each other. (Hint: begins with "p" )

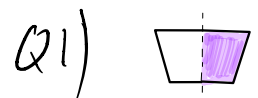
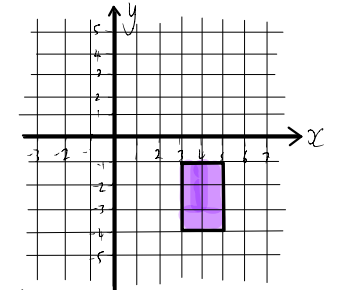
**Question 11**

Reflect the shaded shape in the line  $y = x$ . Label the new shape R.



**Question 12**

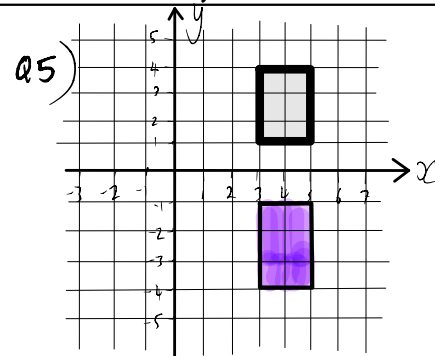
Reflect the shaded shape in the line  $y = -x$ . Label the new shape S.



Q2) B

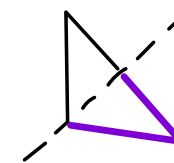
Q3) 3

Q4) the y-axis



Q6) same distance  
symmetry

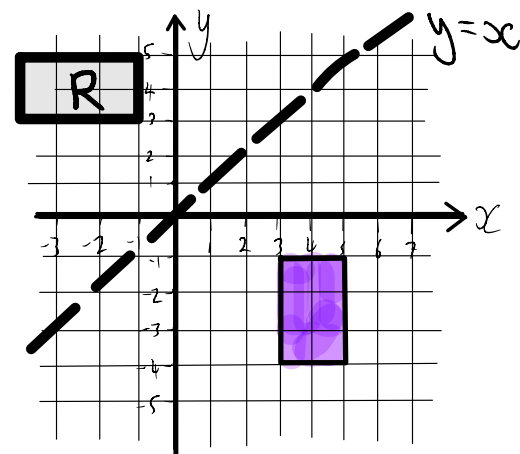
Q7)



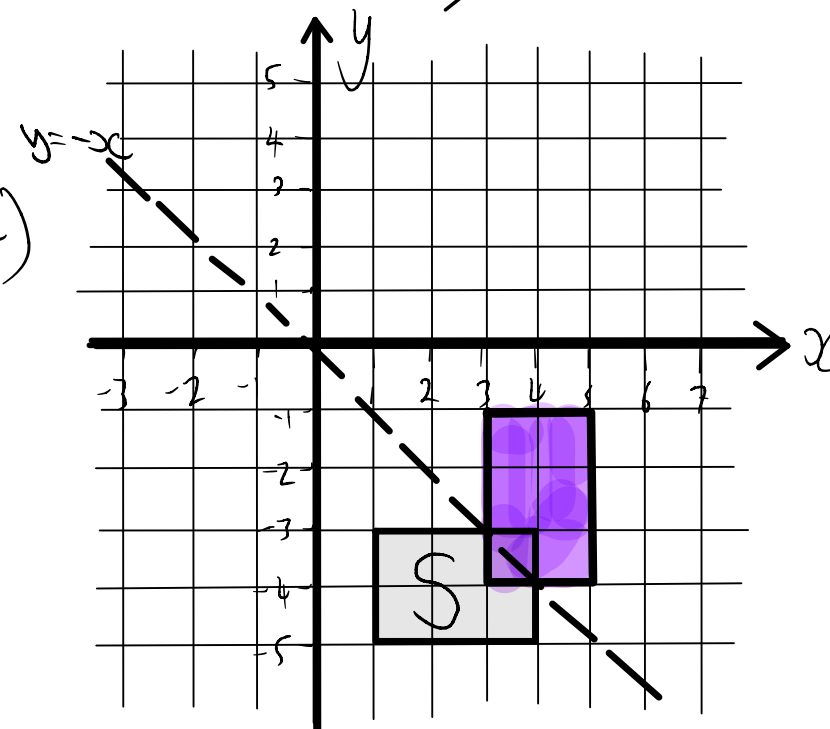
Q8) Sometimes Q9) Only for a regular hexagon.

Q10) perpendicular

Q11)



Q12)



# Year 8 - Reasoning with data...



# The data handling cycle

@whisto\_maths

## What do I need to be able to do?

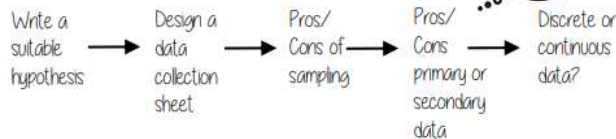
By the end of this unit you should be able to:

- Set up a statistical enquiry
- Design and criticise questionnaires
- Draw and interpret multiple bar charts
- Draw and interpret line graphs
- Represent and interpret grouped quantitative data
- Find and interpret the range
- Compare distributions

## Keywords

- Hypothesis:** an idea or question you want to test
- Sampling:** the group of things you want to use to check your hypothesis
- Primary Data:** data you collect yourself
- Secondary Data:** data you source from elsewhere e.g. the internet/ newspapers/ local statistics
- Discrete Data:** numerical data that can only take set values
- Continuous Data:** numerical data that has an infinite number of values (often seen with height, distance, time)
- Spread:** the distance/ how spread out/ variation of data
- Average:** a measure of central tendency – or the typical value of all the data together
- Proportion:** numerical relationship that compares two things

## Set up a statistical enquiry



### Features of a data collection sheet

Data Title	Tally	Frequency
Grouped or ungrouped categories		Total number of that group observed

## Design and criticise a questionnaire

**The Question** - be clear with the question - don't be too leading/ judgemental

e.g. **How much pocket money do you get a week?**

**Responses** – do you want closed or open responses? – do any options overlap? – Have you an option for all responses?

Zero option →  £0  £0.01 - £2  £2.01 - £4  more than £4 ← More option

**NOTE:** For responses about continuous data include inequalities  $< x \leq$

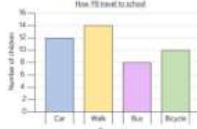
## Pictograms, bar and line charts

### Pictogram

Language	Number of people
French	4
Spanish	4
German	1

- Need to remember a key
- Visually able to identify mode

### Bar Chart



- Gaps between the bars
- Clearly labelled axes
- Scale for the axes
- Title for the bar chart
- Discrete Data

### Line Chart



- Gaps between the lines
- Clearly labelled axes
- Scale for the axes
- Discrete Data

## Draw and interpret Pie Charts

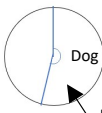
Type of pet	Dog	Cat	Hamster
Frequency	32	25	3

There were 60 people asked in this survey (Total frequency)

$\frac{32}{60}$  "32 out of 60 people had a dog"

This fraction of the 360 degrees represents dogs

$$\frac{32}{60} \times 360 = 192^\circ$$



**Multiple method**  
As 60 goes into 360 – 6 times.  
Each frequency can be multiplied by 6 to find the degrees (proportion of 360)

Use a protractor to draw  
This is 192°

Represents quantitative, discrete data

## Draw and interpret line graphs

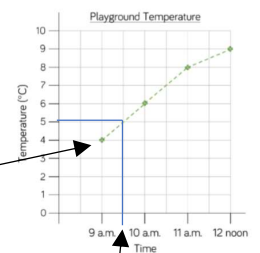
### graphs

- Commonly used to show changing over time
- The points are the recorded information and the lines join the points.

Line graphs do not need to start from 0

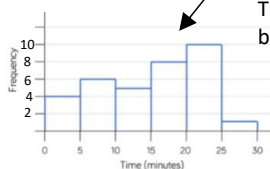
More than one piece of data can be plotted on the same graph to compare data

It is possible to make estimates from the line  
e.g. temperature at 9.30am is 5°C



## Grouped quantitative data

Time (minutes)	Frequency
$0 \leq t < 5$	4
$5 \leq t < 10$	6
$10 \leq t < 15$	5
$15 \leq t < 20$	8
$20 \leq t < 25$	10
$25 \leq t < 30$	1



This is a frequency diagram  
There are no gaps between the bars

Grouping the data is useful if there is a large spread of data to begin with

"More than or equal to 25 and less than 30 minutes"

The use of inequalities shows that this will be a frequency diagram

## Find and interpret the range

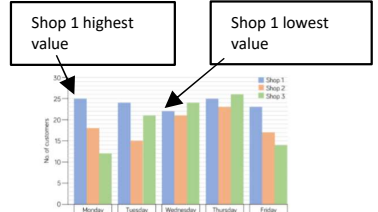
Difference between the biggest and smallest values

The range is a measure of **spread**

A smaller range means there is less variation in the results – it is more consistent data

A range of 0 means all the data is the same value

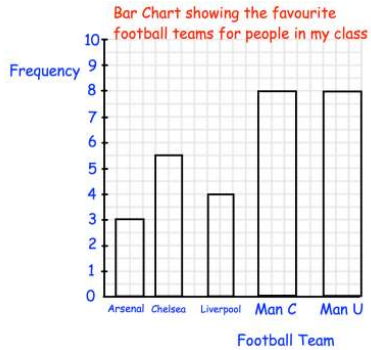
Shop 1 has the smallest range – this indicates it has a more consistent flow of customers each week.



Range of customers =  $25 - 22 = 3$  (Shop 1)



**Question 1**



Write down two things that are wrong with this bar chart.

**Question 2**

Day	Tally	Frequency
Monday		12
Tuesday		
Wednesday		7
Thursday		
Friday		10

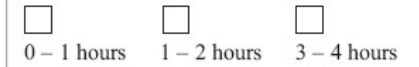
Complete the Tally chart

**Question 3**

Give an example of discrete data.

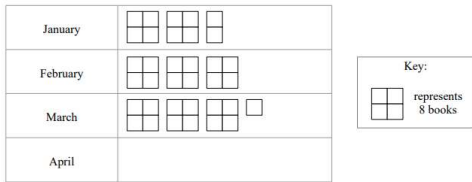
**Question 4**

How much time do you spend playing sport?



Write down 2 things that are wrong with this question

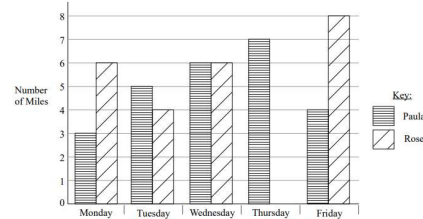
**Question 5**



14 books were sold in April. Show this information on the pictogram.

**Question 6**

The bar chart shows the number of miles Paula and Rose ran from Monday to Friday in a week.

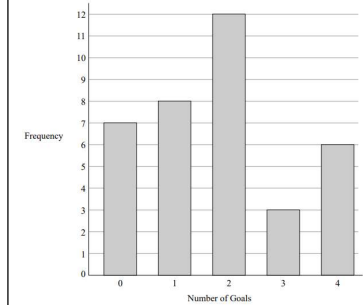


How much further does Rose run than Paula on Monday?

**Question 7**

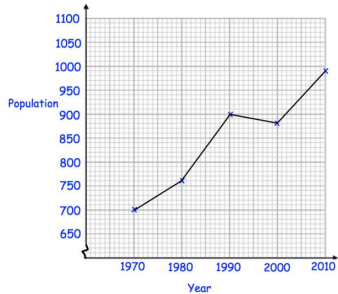
Give an example of continuous data.

**Question 8**



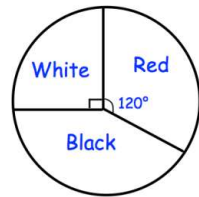
How many goals were scored in total?

**Question 9**



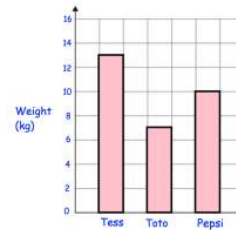
What was the population in 1980?

**Question 10**



A bag contains red, white and black counters. If there are 24 counters in total how many black counters are there?

**Question 11**



The weights of 3 puppies are shown in the chart. What is the range of their weights?

**Question 12**

The table below shows the favourite colours of 30 people. Complete the table to show the angles that would need to be calculated to construct a pie chart.

Colour	black	red	blue	green	pink
Frequency	3	2	2	8	15
Angle					

# Year 8 - Reasoning with data...



## Measures of location

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Understand and use mean, median and mode
- Choose the most appropriate average
- Identify outliers
- Compare distributions using averages and range

### Keywords

**Spread:** the distance/ how spread out/ variation of data

**Average:** a measure of central tendency – or the typical value of all the data together

**Total:** all the data added together

**Frequency:** the number of times the data values occur

**Represent:** something that shows the value of another

**Outlier:** a value that stands apart from the data set

**Consistent:** a set of data that is similar and doesn't change very much

### Mean, Median, Mode

#### The Mean

A measure of average to find the central tendency... a typical value that represents the data

24, 8, 4, 11, 8,

Find the sum of the data (add the values)  $55$

Divide the overall total by how many pieces of data you have  $55 \div 5$

Mean = 11

#### The Median

The value in the centre (in the middle) of the data

24, 8, 4, 11, 8,

Put the data in order 4, 8, 8, 11, 24

Find the value in the middle 4, 8, 8, 11, 24

Median = 8

NOTE: If there is no single middle value find the mean of the two numbers left

#### The Mode (The modal value)

This is the number OR the item that occurs the most (it does not have to be numerical)

24, 8, 4, 11, 8,

This can still be easier if the data is ordered first

Mode = 8

4, 8, 8, 11, 24

### Choosing the appropriate average

The average should be a representative of the data set – so it should be compared to the set as a whole - to check if it is an appropriate average

Here are the weekly wages of a small firm

£240 £240 £240 £240 £240  
£260 £260 £300 £350 £700

Which average best represents the weekly wage?

The Mean = £307

The Median = £250

The Mode = £240

Put the data back into context

Mean/Median – too high (most of this company earn £240)  
Mode is the best average that represents this wage

It is likely that the salaries above £240 are more senior staff members – their salary doesn't represent the average weekly wage of the majority of employers

### Identify outliers

Outliers are values that stand well apart from the rest of the data

Outliers can have a big impact on range and mean.

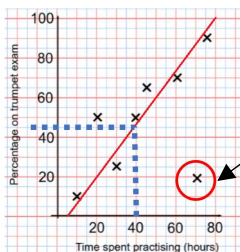
They have less impact on the median

Height in cm  
152 150 142 158 182 151 153 149 156 160 151 144

Sometimes it is best to not use an outlier in calculations

Where an outlier is identified try to give it some context.

This is likely to be a taller member of the group. Could the be an older student or a teacher?



Outliers can also be identified graphically e.g. on scatter graphs

### Comparing distributions

Comparisons should include a statement of average and central tendency, as well as a statement about spread and consistency.

Here are the number of runs scored last month by Lucy and James in cricket matches

Lucy: 45, 32, 37, 41, 48, 35

James: 60, 90, 41, 23, 14, 23

James has two extreme values that have a big impact on the range

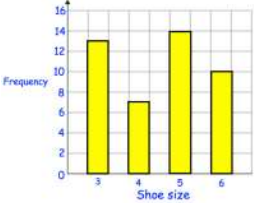
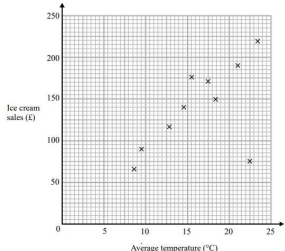
Lucy

Mean: 39.6 (1.d.p), Median: 38. Mode: no mode, Range: 16

James

Mean: 41.8 (1.d.p), Median: 32, Mode: 23, Range: 76

"James is less consistent than Lucy because his scores have a greater range. Lucy performed better on average because her scores have a similar mean and a higher median"

<p>Question 1</p> <p>Work out the mode for the following data:</p> <p>5, 6, 6, 7, 8, 10</p>	<p>Question 2</p> <p>Work out the mode for the following data:</p> <p>5, 5, 7, 7, 7, 8, 8, 9</p>	<p>Question 3</p> <p>Work out the mode for the following data:</p> <p>5, 7, 3, 1, 8, 9, 10, 2</p>	<p>Question 4</p> <p>Work out the mode for the following data:</p> <p>-2, -1, 5, 8, -2, 2, -1, 9, -1, 1, 2, -1</p>	<p>Question 5</p>  <p>What is the modal shoe size?</p>									
<p>Question 6</p> <p>Work out the median for the following data:</p> <p>5, 1, 4, 6, 8</p>	<p>Question 7</p> <p>Work out the median for the following data:</p> <p>6.2, 6.8, 6.6, 7.2, 6.4, 7.4, 5.8</p>	<p>Question 8</p> <p>Work out the median for the following data:</p> <p>9, 8, 6, 6, 6, 7, 1, 2, 6, 8</p>	<p>Question 9</p> <p>Work out the median for the following data:</p> <p>-4, 5, -7, -1, 2, 0, 9</p>	<p>Question 10</p> <p>The height of some footballers are listed below:</p> <p>1.81m, 1.78m, 1.88m, 1.79m, 1.86m, 1.85m, 1.78m, 1.93m</p> <p>Work out the median height.</p>									
<p>Question 11</p> <p>Find the mean of the following data:</p> <p>4, 9, 7, 10, 5</p>	<p>Question 12</p> <p>Find the mean of the following data:</p> <p>3, 2, 1, 3, 2, 2, 1, 3, 1, 2, 3, 2, 1</p>	<p>Question 13</p> <p>Find the mean of the following data:</p> <p>9, -3, -6, 5, 0</p>	<p>Question 14</p> <p>Find the mean of the following data:</p> <p>1.4, 2.8, 2.4, 2.5, 2.8, 3.1, 1.1</p>	<p>Question 15</p> <p>Five houses on a street are sold in 2016. They sell for</p> <p>£175,000 £184,000 £150,000 £201,000 £191,000</p> <p>Calculate the mean price.</p>									
<p>Question 16</p> <p>Find the missing word:</p> <p>The mean, median and mode are known as measures of</p> <p>-----</p>	<p>Question 17</p> <p>Identify the outlier:</p> 	<p>Question 18</p> <p>Which average does outliers affect most?</p>	<p>Question 19</p> <p>Compare the distances travelled by snails and slugs over the duration of ten minutes.</p> <table border="1" data-bbox="1339 1329 1711 1394"> <thead> <tr> <th></th> <th>Median</th> <th>Interquartile range</th> </tr> </thead> <tbody> <tr> <td>Snails</td> <td>7.1 cm</td> <td>3.1 cm</td> </tr> <tr> <td>Slugs</td> <td>9.7 cm</td> <td>4.5 cm</td> </tr> </tbody> </table>		Median	Interquartile range	Snails	7.1 cm	3.1 cm	Slugs	9.7 cm	4.5 cm	<p>Question 20</p> <p>A teacher asks a group of students how much pocket money they receive each week. They respond:</p> <p>£5 £8 £4 £50 £6 £8 £7.50 £10 £8 £7</p> <p>Which average, the median or the mean, is most suitable for this data?</p>
	Median	Interquartile range											
Snails	7.1 cm	3.1 cm											
Slugs	9.7 cm	4.5 cm											

# Straight Line Graphs

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Compare gradients
- Compare intercepts
- Understand and use  $y = mx + c$
- Find the equation of a line from a graph
- Interpret gradient and intercepts of real-life graphs

## Keywords

**Gradient:** the steepness of a line

**Intercept:** where two lines cross. The y-intercept: where the line meets the y-axis.

**Parallel:** two lines that never meet with the same gradient.

**Co-ordinate:** a set of values that show an exact position on a graph.

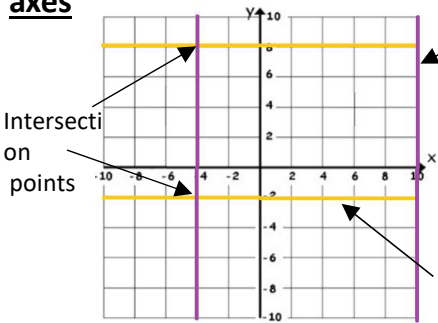
**Linear:** linear graphs (straight line) – linear common difference by addition/subtraction

**Asymptote:** a straight line that a graph will never meet.

**Reciprocal:** a pair of numbers that multiply together to give 1.

**Perpendicular:** two lines that meet at a right angle.

## Lines parallel to the axes



All the points on this line have a x coordinate of 10

'a' can be ANY positive or negative value including 0

Lines parallel to the **y axis** take the form  **$x = a$**  and are **vertical**

Lines parallel to the **x axis** take the form  **$y = a$**  and are **horizontal**

All the points on this line have a y coordinate of -2 e.g. (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

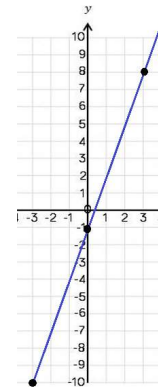
## Plotting $y = mx + c$ graphs

$y = 3x - 1$  → 3 x the x coordinate then -1

x	-3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

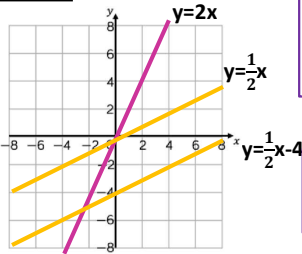
Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

## Compare Gradients

$$y = mx + c$$

The coefficient of x (the number in front of x) tells us the gradient of the line



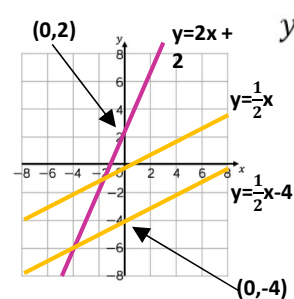
The greater the gradient – the steeper the line

Positive gradients

Negative gradients

Parallel lines have the same gradient

## Compare Intercepts



The value of c is the point at which the line crosses the y-axis. **Y intercept**

The coordinate of a y intercept will always be (0,c)

Lines with the same y-intercept cross in the same place

## $y = mx + c$

The coefficient of x (the number in front of x) tells us the gradient of the line

The value of c is the point at which the line crosses the y-axis. **Y intercept**

y and x are coordinates.

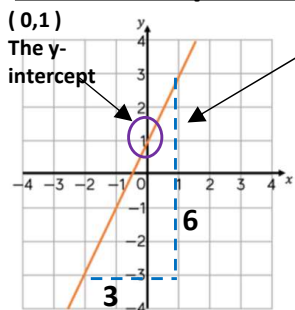
The equation of a line can be rearranged: E.g:

$$y = c + mx$$

$$c = y - mx$$

Identify which coefficient you are identifying or comparing.

## Find the equation from a graph



The Gradient.  $\frac{6}{3} = 2$

$$y = 2x + 1$$

The direction of the line indicates a positive gradient

Positive gradients

Negative gradients

## Real life graphs

A plumber charges a £25 callout fee, and then £12.50 for every hour. Complete the table of values to show the cost of hiring the plumber.

Time (h)	0	1	2	3	8
Cost (£)	£25				£125

The y-intercept shows the minimum charge. The gradient represents the price per mile

In real life graphs like this values will always be positive because they measure distances or objects which cannot be negative.

## Direct Proportion graphs

When you have 0 pens this has 0 cost. The gradient shows the price per pen.

To represent direct proportion the graph must start at the origin.

A box of pens costs £2.30

Complete the table of values to show the cost of buying boxes of pens.

Boxes	0	1	2	3	8
Cost (£)		£2.30			

<p>Question 1</p> <p>Find the gradient of the line <math>2y - 10x = 6</math></p>	<p>Question 2</p> <p>Sketch the graph of <math>y = -x</math></p>	<p>Question 3</p> <p>What is the intercept of the Line <math>y = 10x - 1</math></p>	<p>Question 4</p> <p>Does the point <math>(3, -19)</math> lie on the line <math>Y = -6x - 2</math></p>
<p>Question 5</p> <p>What is the y-intercept of the line <math>y = 3x + 7</math>?</p>	<p>Question 6</p> <p>What is the intercept <math>Y = 2x - 1</math></p>	<p>Question 7</p> <p>The point <math>(4, y)</math> lies on the line <math>y = 2x + 10</math>. Find <math>y</math></p>	<p>Question 8</p> <p>Find the value of <math>3x + 1</math> When <math>x = -1</math></p>
<p>Question 9</p> <p>Sketch the graph of <math>y = 2x</math></p>	<p>Question 10</p> <p>What is the gradient <math>Y = 2x - 1</math></p>	<p>Question 11</p> <p>Sketch the graph of <math>y = 2</math></p>	<p>Question 12</p> <p>Find an equation parallel to the line. <math>Y = 2x - 1</math></p>
<p>Question 13</p> <p>Sketch the graph of <math>x = 2</math></p>	<p>Question 14</p> <p>What is the gradient? <math>Y = 8x - 10</math></p>	<p>Question 15</p> <p>Sketch the graph of <math>y = -x + 2</math></p>	<p>Question 16</p> <p>What is this line on the graph</p>
<p>Question 17</p> <p>The point <math>(3, y)</math> lies on the line <math>y = 4x - 7</math>. Find <math>y</math></p>	<p>Question 18</p> <p>What is the intercept <math>Y = 6 - x</math></p>	<p>Question 19</p> <p>Find the value of <math>3x - 1</math> when <math>x = 1</math></p>	<p>Question 20</p> <p>Sketch the line <math>X + y = 4</math></p>



# Year 9 – using number...

## Indices & Roots

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### What do I need to be able to do?

By the end of this unit you should be able to:

- Identify square and cube numbers
- Calculate higher powers and roots
- Understand powers of 10 and standard form
- Know the addition and subtraction rule for indices
- Understand power zero and negative indices
- Calculate with numbers in standard form

### Keywords

- Standard (index) Form:** A system of writing very big or very small numbers
- Commutative:** an operation is commutative if changing the order does not change the result.
- Base:** The number that gets multiplied by a power
- Power:** The exponent – or the number that tells you how many times to use the number in multiplication
- Exponent:** The power – or the number that tells you how many times to use the number in multiplication
- Indices:** The power or the exponent.
- Negative:** A value below zero..
- Coefficient:** The number used to multiply a variable

### Square and cube numbers

#### Square numbers

1, 4, 9, 16...

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

$$(2 \times 2 \times 3) \times (2 \times 2 \times 3)$$

Prime factors can find square roots

$$\sqrt{144} = 12$$

#### Cube

1, 8, 27, 64, 125...

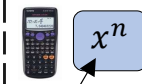
$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

$$(2 \times 3) \times (2 \times 3) \times (2 \times 3)$$

$$6 \times 6 \times 6$$

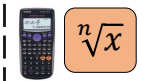
$$\sqrt[3]{216} = 6$$

### Higher powers and roots



x – the base number.

n – power (number of times multiplied by itself)



Finding the nth root of any value

### Standard form

Any number between 1 and less than 10

#### Example

$$3.2 \times 10^4 = 3.2 \times 10 \times 10 \times 10 \times 10 = 32000$$

Any integer

$$A \times 10^n$$

0.001

$$1 \times \frac{1}{1000}$$

$$1 \times 10^{-3}$$

10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
$10^1$	$10^0$	$10^{-1}$	$10^{-2}$	$10^{-3}$
10	1	0.1	0.01	0.001

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

Numbers in standard form with negative powers will be less than 1

$$3.2 \times 10^{-4} = 3.2 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = 0.00032$$

### Other mental strategies for square roots

$$\begin{aligned} \sqrt{810000} &= \sqrt{81} \times \sqrt{10000} \\ &= 9 \times 100 \\ &= 900 \end{aligned}$$

### Addition/ Subtraction Laws

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

### Zero and negative indices

$$x^0 = 1$$

$$\begin{aligned} \frac{a^6}{a^6} &= a^6 \div a^6 \\ &= a^{6-6} = a^0 = 1 \end{aligned}$$

Negative indices do not indicate negative solutions

$$\begin{aligned} 2^2 &= 4 \\ 2^1 &= 2 \\ 2^0 &= 1 \\ 2^{-1} &= \frac{1}{2} \\ 2^{-2} &= \frac{1}{4} \end{aligned}$$

Looking at the sequence can help to understand negative powers

### Powers of powers

$$(x^a)^b = x^{ab}$$

$$(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$$

The same base and power is repeated. Use the addition law for indices

$$(2^3)^4 = 2^{12} \leftarrow a \times b = 3 \times 4 = 12$$

#### NOTICE the difference

$$(2x^3)^4 = 2x^3 \times 2x^3 \times 2x^3 \times 2x^3$$

The addition law applies ONLY to the powers. The integers still need to be multiplied

$$(2x^3)^4 = 16x^{12}$$

### Standard form calculations

#### Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard form at the end.

#### Method 1

$$= 600000 + 800000$$

$$= 1400000$$

$$= 1.4 \times 10^5$$

#### Multiplication and division

$$1.5 \times 10^5$$

$$0.3 \times 10^3$$

$$(1.5 \times 10^5) \div (0.3 \times 10^3)$$

$$1.5 \div 0.3 \times 10^5 \div 10^3$$

$$= 5 \times 10^2$$

#### Method 2

$$= (6 + 8) \times 10^5$$

$$= 14 \times 10^5$$

$$= 1.4 \times 10^1 \times 10^5$$

$$= 1.4 \times 10^6$$

This is not the final answer

Division questions can look like this

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations

<b>Question 1</b> Evaluate $6^0$	<b>Question 2</b> Work out $(2 \times 10^4) \times (7.8 \times 10^4)$	<b>Question 3</b> Express the reciprocal of 16 as a fraction	<b>Question 4</b> Simplify $(7^4)^9$
<b>Question 5</b> Calculate $\sqrt{81}$	<b>Question 6</b> Work out $(1 \times 10^9) \div (5 \times 10^5)$	<b>Question 7</b> Simplify $2^3 \times 2^6$	<b>Question 8</b> Simplify $(2^3)^5$
<b>Question 9</b> Evaluate $2^{-2}$	<b>Question 10</b> Work out $(4 \times 10^2) \times (8.5 \times 10^2)$	<b>Question 11</b> Evaluate $5^1$	<b>Question 12</b> Calculate $\sqrt[3]{64}$
<b>Question 13</b> Simplify $(4^2)^6$	<b>Question 14</b> Express the reciprocal of $\frac{3}{10}$ as a fraction.	<b>Question 15</b> Work out $(8.1 \times 10^8) \div (9 \times 10^2)$	<b>Question 16</b> Calculate $\sqrt{169}$
<b>Question 17</b> Work out $(4 \times 10^6) \times (1.5 \times 10^2)$	<b>Question 18</b> Express the reciprocal of 2 as a decimal.	<b>Question 19</b> Simplify $3^3 \div 3^6$	<b>Question 20</b> Work out $(9 \times 10^7) \div (6 \times 10^4)$

1,  $1.56 \times 10^9$ ,  $\frac{1}{6}$ ,  $7^{36}$ , 9,  $2 \times 10^3$ ,  $2^9$ ,  $2^{15}$ ,  $\frac{1}{4}$ ,  $3.4 \times 10^5$ , 5, 4,  $4^{12}$ ,  $\frac{10}{3}$ ,  $9 \times 10^5$ , 13,  $6 \times 10^8$ ,  $3^{-3}$ ,  $1.5 \times 10^3$



## Solving ratio & proportion problems

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems with direct proportion
- Use conversion graphs
- Solve problems with inverse proportion
- Solve ratio problems
- Solve 'best buy' problems

### Keywords

**Proportion:** a comparison between two numbers

**Ratio:** a ratio shows the relative size of two variables

**Direct proportion:** as one variable is multiplied by a scale factor the other variable is multiplied by the same scale factor.

**Inverse proportion:** as one variable is multiplied by a scale factor the other is divided by the same scale factor.

### Direct Proportion

As one variable changes the other changes at the same rate.



4 cans of pop = £2.40

4 cans of pop = £2.40

2 cans of pop = £1.20

This multiplier is the same  
In the same way that this would be for ratio

This is a multiplicative change

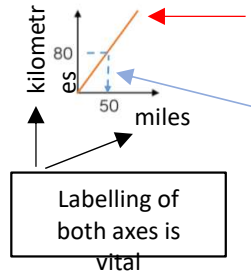
4 cans of pop = £2.40

12 cans of pop = £7.20

Sometimes this is easiest if you work out how much one unit is worth first  
e.g. 1 can of pop = £0.60

### Conversion Graphs

Compare two variables



This is always a straight line because as one variable increases so does the other at the same rate

To make conversions between units you need to find the point to compare – then find the associated point by using your graph.

Using a ruler helps for accuracy  
Showing your conversion lines help as a "check" for solutions

### Inverse Proportion

As one variable is multiplied by a scale factor the other is divided by the same scale factor

Examples of inversely proportional relationships

Time taken to fill a pool and the number of taps running.

Time taken to paint a room and the number of workers

T is inversely proportional to G. When T=2 then G=20

T	1	2	8
G	40	20	5

Annotations: 1 to 2 is x2, 2 to 8 is x4, 40 to 20 is ÷2, 20 to 5 is ÷4

### Best Buys

Have a directly proportional relationship

To calculate best buys you need to be able to compare the cost of one unit or units of equal amounts



Shop A: 4 cans for £1.20

Shop B: 3 cans for 93p

£1.20 ÷ 4

£0.93 ÷ 3

Cost per item

1 can is £0.30  
Or 30p

1 can is £0.31  
Or 31p

Shop A is the best value as it is 1p cheaper per can of pop



Shop A: 4 cans for £1.20

Shop B: 3 cans for 93p

4 ÷ £1.20

3 ÷ £0.93

Cost per pound

£1 buys 3.333 cans of pop

£1 buys 3.23 cans of pop

Shop A is still shown as being the best value but pay attention to the unit you are calculating, per item or per pound.

Best value is the most product for the lowest price per unit

### Sharing a whole into a given ratio

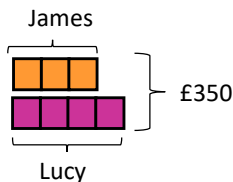


James and Lucy share £350 in the ratio 3:4. Work out how much each person earns

Model the Question

James:  
Lucy:

3 : 4



£350 ÷ 7 = £50

□ = one part = £50

Find the value of one part

7 parts to share between

(3 James, 4 Lucy)

Put back into the question

James: Lucy

James = 3 x £50 = £150

Lucy = 4 x £50 = £200

3 : 4  
x 50      x 50  
£150 : £200

### Finding a value given 1:n (or n:1)



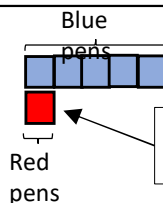
Inside a box are blue and red pens in the ratio 5:1.

If there are 10 red pens how many blue pens are there?

Model the Question

Blue : Red

5 : 1



One unit = 10 pens

Put back into the question

Blue : Red

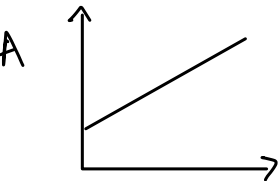
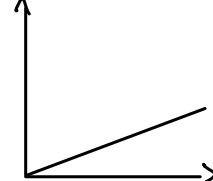
5 : 1  
x 10      x 10  
50 : 10

Blue pens = 5 x 10 = 50



Red pens = 1 x 10 = 10

There are 50 Blue Pens

<p><b>Question 1</b> Simplify the ratio <math>27 : 45</math></p>	<p><b>Question 2</b> Write the ratio in the form <math>1:n</math> <math>12 : 18</math></p>	<p><b>Question 3</b> Divide £24 in the ratio <math>1:3</math></p>	<p><b>Question 4</b> 3 boxes of pencils contain 150 pencils altogether. How many pencils are there in 9 boxes?</p>
<p><b>Question 5</b> Which is the better value for money? A : 2 erasers for 57p B : 3 erasers for 90p</p>	<p><b>Question 6</b> It takes 8 people 12 hours to build a wall. How long would it take 16 people?</p>	<p><b>Question 7</b> Jack runs 1.6km in 2 hours. What is his speed in km/h?</p>	<p><b>Question 8</b> Work out the density in <math>\text{kg/m}^3</math> for a mass of 640kg and a volume of <math>0.8\text{m}^3</math>.</p>
<p><b>Question 9</b> Sally drives at 34mph for 0.8 hours. How far does she travel in miles?</p>	<p><b>Question 10</b> 3 boxes of pencils contain 150 pencils altogether. How many pencils are there in 27 boxes?</p>	<p><b>Question 11</b> Work out the volume in <math>\text{m}^3</math> for a density of <math>2.5\text{kg/m}^3</math> and a mass of 0.675kg.</p>	<p><b>Question 12</b> Which is the better value for money? A : 2kg of potatoes for £1.60 B : 7kg of potatoes for £5.53</p>
<p><b>Question 13</b> Work out the mass in kg for a density of <math>9\text{kg/m}^3</math> and a volume of <math>72\text{m}^3</math>.</p>	<p><b>Question 14</b> Which graph shows a direct proportion relationship? A  or B </p>	<p><b>Question 15</b> Simplify the ratio <math>120 : 720</math></p>	<p><b>Question 16</b> Divide 96 in the ratio <math>7:1:4</math></p>
<p><b>Question 17</b> Write the ratio in the form <math>1:n</math> <math>2 : 5</math></p>	<p><b>Question 18</b> It takes 8 people 12 hours to build a wall. How long would it take 1 person?</p>	<p><b>Question 19</b> 3 boxes of pencils contain 150 pencils altogether. How many boxes do you need to have 1500 pencils?</p>	<p><b>Question 20</b> Tom travels 3.8km at 20km/h. For how long was he travelling? (in hours)</p>

# Year 9 - algebraic techniques...



## Brackets, Equations & Inequalities

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Form Expressions
- Expand and factorise single brackets
- Form and solve equations
- Solve equations with brackets
- Represent inequalities
- Form and solve inequalities

### Keywords

- Simplify:** grouping and combining similar terms
- Substitute:** replace a variable with a numerical value
- Equivalent:** something of equal value
- Coefficient:** a number used to multiply a variable
- Product:** multiply terms
- Highest Common Factor (HCF):** the biggest factor (or number that multiplies to give a term)
- Inequality:** an inequality compares two values showing if one is greater than, less than or equal to another

### Form expressions

For unknown variables, a letter is normally used in its place

More than – **ADD**  
Less than/ difference – **SUBTRACT**  
e.g. 4 more than t →  $t + 4$   
8 less than k →  $k - 8$

Only similar terms can be grouped together

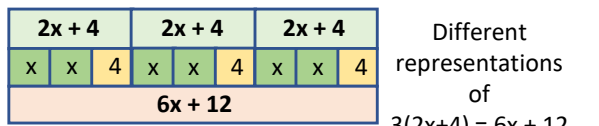
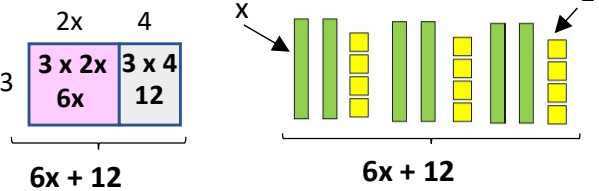
e.g. Find the perimeter of this shape (Perimeter = length around outside of shape)  
 $2t + 1 + t + 2t + 1 + t + 2t + 1 \rightarrow 6t + 2$

### Directed numbers

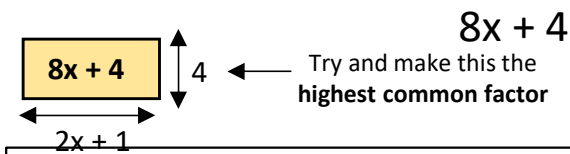
$++ \rightarrow +$   
 $-- \rightarrow +$   
 $+- \rightarrow -$   
 $-+ \rightarrow -$

e.g.  $a = -5$  and  $b = 2$   
 $a^2 = a \times a = -5 \times -5 = 25$   
 $b + a = 2 + -5 = -3$

### Multiply single brackets $3(2x + 4)$



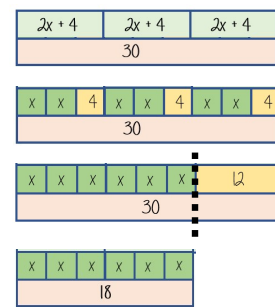
### Factorise into a single bracket



The two values **multiply** together (also the area) of the rectangle

Note:  
 $8x + 4 \equiv 4(2x + 1)$   
 $8x + 4 \equiv 2(4x + 2)$   
This is factorised but the HCF has not been used

### Solve equations with brackets $3(2x + 4) = 30$



$3(2x + 4) = 30$   
Expand the brackets  
 $6x + 12 = 30$   
 $-12 \quad -12$   
 $6x = 18$   
 $\div 6 \quad \div 6$   
 $x = 3$

Substitute to check your answer.  
This could be negative or a fraction or decimal

### Simple Inequalities

$<$  less than       $\leq$  Less than or equal to  
 $>$  More than       $\geq$  More than or equal to

$x < 10$   
Say this out loud "x is a value less than 10"

$10 > x$   
Say this out loud "10 is more than the value"

Note:  
 $x < 10$  and  $10 > x$  represent the same values

$x + 2 \leq 20$   
"my value + 2 is less than or equal to 20"

$x \leq 18$   
The biggest the value can be is 18

### Form and solve inequalities

Two more than treble my number is greater than 11

Find the possible range of values

**Form**  
 $x \rightarrow x3 \rightarrow +2 \rightarrow 11$   
 $3x + 2 > 11$

**Solve**  
 $x \leftarrow \div 3 \leftarrow -2 \leftarrow 11$   
 $x > 3$

**Check**  
This would suggest any value bigger than 3 satisfies the statement  
 $3 \times 3 + 2 = 11 \checkmark$        $10 \times 3 + 2 = 32 \checkmark$

### Algebraic constructs

- Expression**  
A sentence with a minimum of two numbers and one maths operation
- Equation**  
A statement that two things are equal
- Term**  
A single number or variable
- Identity**  
An equation where both sides have variables that cause the same answer includes  $\equiv$
- Formula**  
A rule written with all mathematical symbols  
E.g. area of a rectangle  $A = b \times h$

<b>Question 1</b> If $a = 12$ , Work out $2a$	<b>Question 2</b> Simplify $3a + 2a - 4a$	<b>Question 3</b> Simplify $3a \times a$	<b>Question 4</b> Factorise fully $3x + 12$
<b>Question 5</b> Simplify $4a \times 2b$	<b>Question 6</b> Expand $2(x+3)$	<b>Question 7</b> Expand and Simplify $(x-3)(x+6)$	<b>Question 8</b> Simplify $6x + 3y + 4x - y$
<b>Question 9</b> Factorise fully $3x^2 - 6x$	<b>Question 10</b> If $a = 12$ and $b = 3$ Work out $a \times b$	<b>Question 11</b> Factorise fully $15x^3 + 80x$	<b>Question 12</b> Expand $4(x-3)$
<b>Question 13</b> Simplify $5p^2 + 10p + 4p^2 - 2p$	<b>Question 14</b> Expand and simplify $5(x+1) + 3(x-3)$	<b>Question 15</b> Simplify $-v \times v$	<b>Question 16</b> If $a = 12$ and $b = 3$ , Work out $a^2 - 2b$
<b>Question 17</b> Expand and Simplify $(2x+5)(x-3)$	<b>Question 18</b> If $a = 7$ and $b = -3$ Work out $3a - 2b$	<b>Question 19</b> Expand and simplify $7(x+4) - 3(x-3)$	<b>Question 20</b> Simplify $4t \times 2g$

# Year 9 – Reasoning with algebra...

## Forming and Solving Equations

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Solve inequalities with negative numbers
- Solve equations with unknowns on both sides
- Solve inequalities with unknowns on both sides
- Substitute into formulae and equations
- Rearrange formulae

### Keywords

- Inequality:** an inequality compares two values showing if one is greater than, less than or equal to another
- Variable:** a quantity that may change within the context of the problem
- Rearrange:** Change the order
- Inverse operation:** the operation that reverses the action
- Substitute:** replace a variable with a numerical value
- Solve:** find a numerical value that satisfies an equation

### Solve equations with brackets

$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$

$-12 \quad -12$

$6x = 18$

$\div 6 \quad \div 6 \quad x = 3$

### Form and solve inequalities

Two more than treble my number is greater than 11

Find the possible range of values

$3x + 2 > 11$

**Solve**

$x \leftarrow \div 3 \leftarrow -2 \leftarrow 11$

$x > 3$

### Inequalities with negatives

**Method 1:** Make x positive first

$2 - 3x > 17$

$+3x \quad +3x$

$2 > 17 + 3x$

$-17 \quad -17$

$-15 > 3x$

$\div 3 \quad \div 3$

$-5 > x$

x is true for any value smaller than -5

**CHECK IT!**

$2 - 3(-6) = 20$

**TRUE/ CORRECT**

Small er  $\leftarrow$   $\rightarrow$  Bigger

$-7 \quad -5 \quad -3$

### Equations with unknown on both sides

$4x + 5 = 3x + 24$

$-3x \quad -3x$

$x + 5 = 24$

$-5 \quad -5$

$x = 19$

### Inequalities with unknown on both sides

Solving inequalities has the same method as equations

**Check it!**

$5(x + 4) < 3(x + 2)$

$5x + 20 < 3x + 6$

$2x + 20 < 6$

$2x < -14$

$x < -7$

$5(-8 + 4) < 3(-8 + 2)$

$5(-4) < 3(-6)$

$-20 < -18$

$20$  IS smaller than  $-18$

**Method 2:** Keep the negative x

$2 - 3x > 17$

$-2 \quad -2$

$-3x > 15$

$\div -3 \quad \div -3$

$x > -5$

x is true for any value bigger than -5

**This cannot be true...**

$x < -5$  When you multiply or divide by a negative you need to reverse the inequality

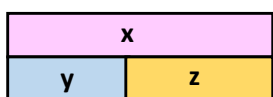
### Formulae and Equations

**Formulae** – all expressed in symbols

**Substitute** in values

**Equations** – include numbers and can be solved

### Rearranging Formulae (one step)



$x = y + z$

Rearrange to make y the **y = x - z** subject.

$y \rightarrow +z \rightarrow x$

$y \leftarrow -z \leftarrow x$

Using inverse operations or fact families will guide you through rearranging formulae

Rearranging can also be checked by substitution. Language of rearranging...

Make XXX the subject

Change the subject

Rearrange

### Rearranging Formulae (two step)

In an equation (find x)

$4x - 3 = 9$

$+3 \quad +3$

$4x = 12$

$\div 4 \quad \div 4$

$x = 3$

In a formula (make x the subject)  $s = a$

$+s \quad +s$

$xy = a + s$

$\div y \quad \div y$

$x = \frac{a + s}{y}$

The steps are the same for solving and rearranging

Rearranging is often needed when using  $y = mx + c$

e.g. Find the gradient of the line  $2y - 4x = 9$

**Make y the subject first**

$y = \frac{4x + 9}{2}$

Gradient =  $\frac{4}{2} = 2$

**Question 1**

Solve

$$4x = 12$$

**Question 2**

Solve

$$3(x+4) = 27$$

**Question 3**

$$\text{Solve } x+6 > 2$$

**Question 4**

An approximate formula for converting Celsius to Fahrenheit is

$$F = 2C + 30$$

Find F when  $C = 17$ **Question 5**

Solve

$$5x - 8 > 56$$

**Question 6**

$$\text{Solve } w - 12 \leq 2$$

**Question 7**

$$\text{Solve } p + 11 = 23$$

**Question 8**

Solve

$$5(5x+1) = 105$$

**Question 9**

Solve

$$6(3x+6) = 4(5x-2)$$

**Question 10**

$$\text{Solve } 2a+5 = 13$$

**Question 11**

Solve

$$2p \geq 8$$

**Question 12**

Solve

$$4x - 7 > 35$$

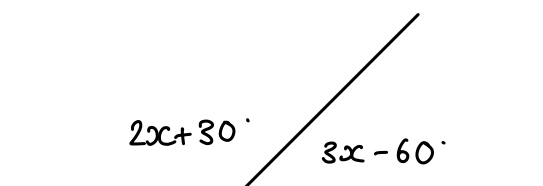
**Question 13**

An approximate formula for converting Celsius to Fahrenheit is

$$F = 2C + 30$$

Find C when  $F = 78$ **Question 14**

Use angle facts to solve:

**Question 15**

Solve

$$3(2x+1) + 2(4x+2) = 35$$

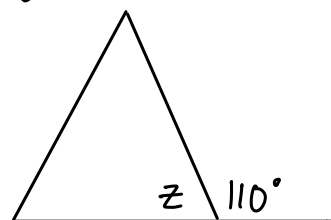
**Question 16**

Solve

$$3b - 4 = 17$$

**Question 17**

Use angle facts to solve:

**Question 18**

Solve

$$5d - 1 = 9$$

**Question 19**

Solve

$$4(x+2) = 40$$

**Question 20**

Solve

$$40 - 3x \leq 10$$



## Testing conjectures

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

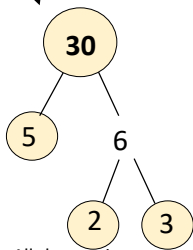
- Use factors, multiples and primes
- Reason True or False
- Reason Always, sometimes never true
- Show that reasoning
- Make conjectures about number
- Expand binomials
- Make conjectures with algebra
- Explore the 100 grid

### Keywords

- Multiples:** found by multiplying any number by positive integers
- Factor:** integers that multiply together to get another number.
- Prime:** an integer with only 2 factors.
- HCF:** highest common factor (biggest factor two or more numbers share)
- LCM:** lowest common multiple (the first time the times table of two or more numbers match)
- Verify:** the process of making sure a solution is correct
- Proof:** logical mathematical arguments used to show the truth of a statement
- Binomial:** a polynomial with two terms
- Quadratic:** a polynomial with four terms (often simplified to three terms)

### Factors, Multiples and Primes

Multiplication part-whole models



All three prime factor trees represent the same decomposition

#### HCF – Highest common factor

##### HCF of 18 and 30

**18** 1, 2, 3, 6, 9, 18

**30** 1, 2, 3, 5, 6, 10, 15, 30

Common factors are factors two or more numbers share

#### LCM – Lowest common multiple

##### LCM of 9 and 12

**9** 9, 18, 27, 36, 45, 54

**12** 12, 24, 36, 48, 60

Common multiples are multiples two or more numbers share



### True or False?

#### Conjecture

A pattern that is noticed for many cases

1, 2, 4, ...  
The numbers in the sequence are doubling each time.

#### Counterexamples



This sequence isn't doubling it is adding 2 each time

Only **one** counterexample is needed to disprove a conjecture

### Always, Sometimes, Never true.

**Always** Every value always supports the statement

**Sometimes** Examples show the statement being true and counter examples to show when it is false.

**Never** No example supports the statement

- Examples to try
- 0 and 1
  - Fractions
  - Negative numbers

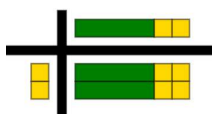
### Show that

- Numerical verification** Show the stages to a solution with numerical values
- Algebraic verification** Show algebraic properties of the solution
- Proof** You may want to use pictorial images to support this. Simple proofs using algebra

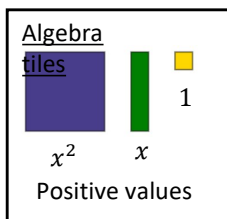
Compare the left hand side of an equation with the right hand side – are they the same or different?

### Expanding binomials

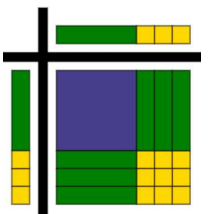
$$2(x + 2) \equiv 2x + 4$$



Algebra tiles can represent a binomial expansion. Has two terms



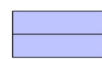
$$(x + 3)(x + 3) \equiv x^2 + 6x + 9$$



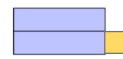
This is a quadratic. It has four terms which simplified to three terms

The order of the binomial has no impact on the outcome. e.g.  $(x + 3)(3 + x)$

### Conjectures



**Even**  
 $(2n)$   
Multiple of 2



**Odd**  
 $(2n + 1)$   
One more than any even

Use numerical verification first. Use pictorial verification – the representations of numbers of odd and even

### Exploring the 100 square

In terms of 'n' is used to make generalisations about relationships between numbers. Positions of numbers in relation to n form expressions.

E.g. one space to the right of n  
 $n + 1$   
E.g. One row below n  
 $n + 10$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The size of the grid for generalisation changes the relationship statements



<b>Question 1</b> List the first six multiples of 8	<b>Question 2</b> Express 80 as a product of prime factors	<b>Question 3</b> Scott rolls a fair dice numbered 1-6 What is the probability that it lands on a prime number?	<b>Question 4</b> True or False : All prime numbers are odd
<b>Question 5</b> $450 = 2 \times 3 \times 3 \times 5 \times 5$ Use this fact to write 900 as a product of prime factors	<b>Question 6</b> Liam says "the angles in this triangle are $80^\circ$ , $80^\circ$ and $70^\circ$ " How do you know that he is wrong?	<b>Question 7</b> List the first six multiples of 15	<b>Question 8</b> Show that $x=6$ is a solution to the equation $7x + 43 = 85$
<b>Question 9</b> True or False : 10 is a factor of 5	<b>Question 10</b> $450 = 2 \times 3 \times 3 \times 5 \times 5$ Use this fact to write 225 as a product of prime factors	<b>Question 11</b> Express 68 as a product of prime factors	<b>Question 12</b> True or False : The angles in a triangle sum to $180^\circ$

Answers : 1. 8, 16, 24, 32, 40, 48

2.  $80 = 2 \times 2 \times 2 \times 2 \times 5 = 2^4 \times 5$

3.  $\frac{3}{6}$  or  $\frac{1}{2}$

4. False. 2 is the only even prime number.

5.  $900 = 2 \times 2 \times 3 \times 3 \times 5 \times 5$

6.  $80 + 80 + 70 = 230^\circ$ , angles in a triangle sum to  $180^\circ$

7. 15, 30, 45, 60, 75, 90

8.  $7 \times 6 = 42$ ,  $42 + 43 = 85$

9. False. 10 is a multiple of 5

10.  $225 = 3 \times 3 \times 5 \times 5$

11.  $68 = 2 \times 2 \times 17 = 2^2 \times 17$

12. True

# 3D Shapes

@whisto\_maths

## What do I need to be able to do?

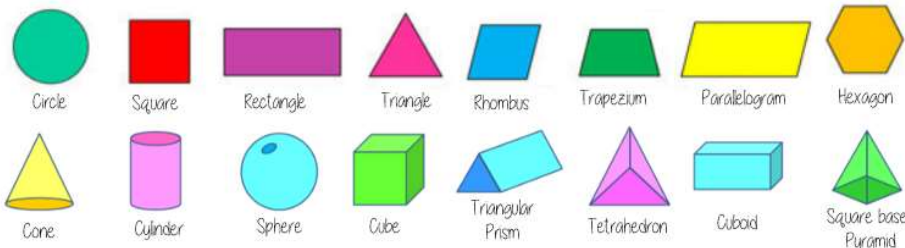
By the end of this unit you should be able to:

- Name 2D & 3D shapes
- Recognise Prisms
- Sketch and recognise nets
- Draw plans and elevations
- Find areas of 2D shapes
- Find Surface area for cubes, cuboids, triangular prisms and cylinders
- Find the volume of 3D shapes

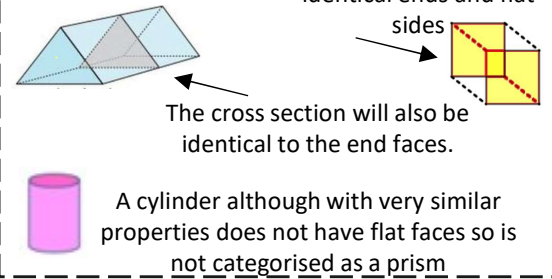
## Keywords

- 2D:** two dimensions to the shape e.g. length and width  
**3D:** three dimensions to the shape e.g. length, width and height  
**Vertex:** a point where two or more line segments meet  
**Edge:** a line on the boundary joining two vertex  
**Face:** a flat surface on a solid object  
**Cross-section:** a view inside a solid shape made by cutting through it  
**Plan:** a drawing of something when drawn from above (sometimes birds eye view)  
**Perspective:** a way to give illustration of a 3D shape when drawn on a flat surface.

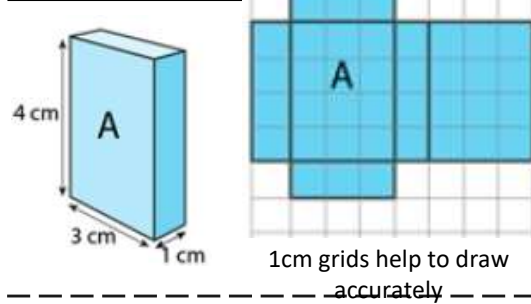
## Name 2D & 3D shapes



## Recognise prisms

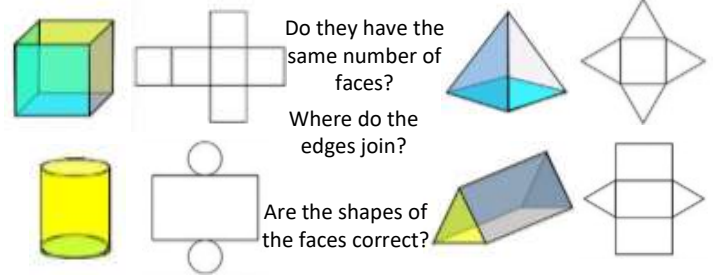


## Nets of cuboid

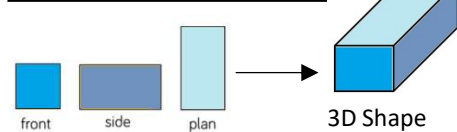


Visualise the folding of the net.  
Will it make the cuboid with all sides touching

## Sketch and recognise nets

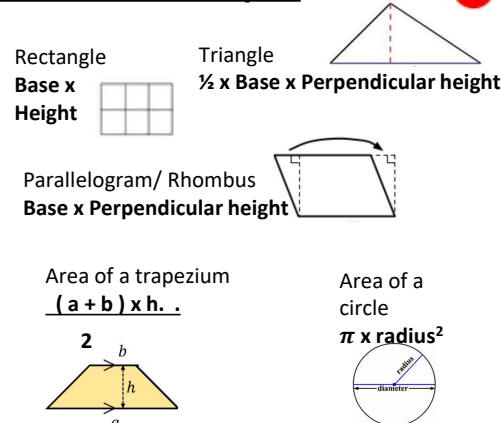


## Plans and elevation

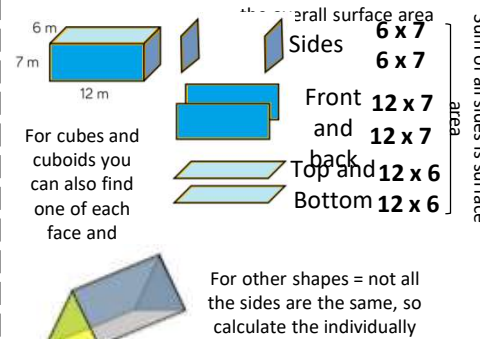


The direction you are considering the shape from determines the front and side views

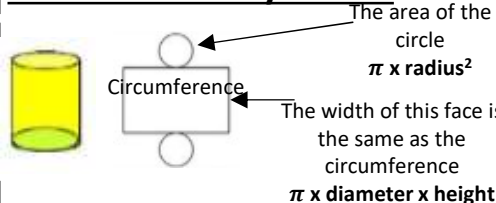
## Area of 2D shapes



## Surface area



## Surface area - cylinders



$$2 \times \pi \times \text{radius}^2 + \pi \times \text{diameter} \times \text{height}$$

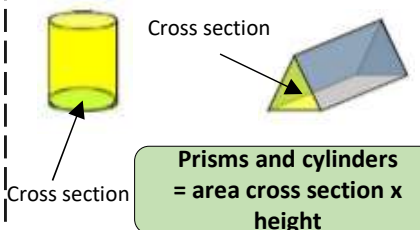
## Volumes

Volume is the 3D space it takes up – also known as capacity if using liquids to fill the space

**Counting cubes**  
Some 3D shape volumes can be calculated by counting the number of cubes that fit inside the shape.

$$\text{Cubes/ Cuboids} = \text{base} \times \text{width} \times \text{height}$$

Remember multiplication is commutative

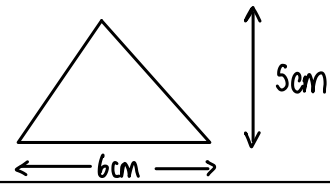


Height can also be described as depth

Areas – square units  
Volumes – cube units  
Areas and volumes can be left in terms of  $\pi$

**Question 1**

Find the area of this shape:



**Question 2**

The height of a cube is 5cm.

Find the volume of the cube.

**Question 3**

Find the circumference of a circle with radius 9cm. Give your answer in terms of  $\pi$ .

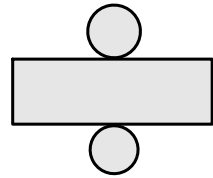
**Question 4**

The volume of a cube is  $343\text{cm}^3$ .

Find the height of the cube.

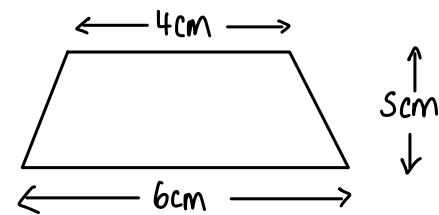
**Question 5**

This is a net for which 3D shape?



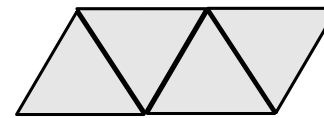
**Question 6**

Find the area of this shape:



**Question 7**

This is a net for which 3D shape?

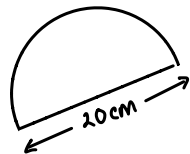


**Question 8**

Find the circumference of a circle with diameter 5cm. Give your answer in terms of  $\pi$ .

**Question 9**

Find the area of this shape. Give your answer in terms of  $\pi$ :

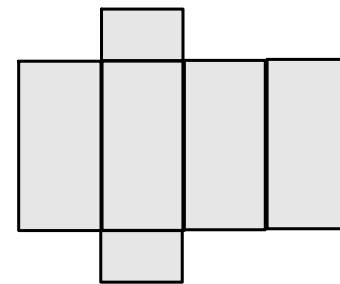


**Question 10**

Find the volume of the cuboid with length 10cm, width 4cm and height 5cm.

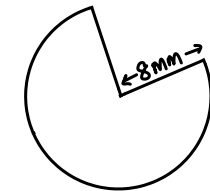
**Question 11**

This is a net for which 3D shape?



**Question 12**

Find the area of this shape. Give your answer in terms of  $\pi$ :



**Question 13**

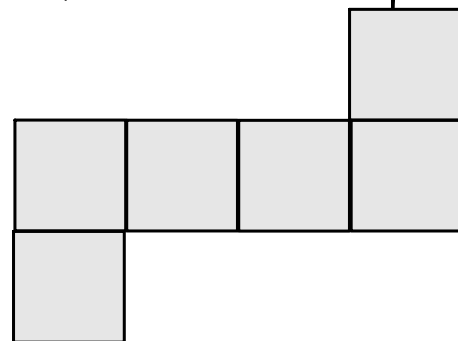
The volume of a cuboid is  $192\text{cm}^3$ .

The length is 8cm and the width is 4cm.

Calculate the height.

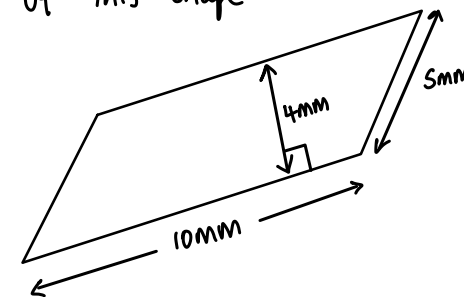
**Question 14**

This is a net for which 3D shape?



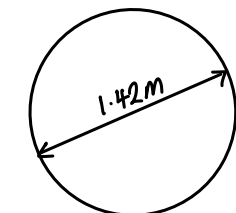
**Question 15**

Find the area of this shape:



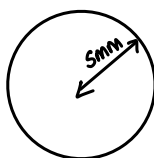
**Question 16**

Find the area of this shape. Give your answer in terms of  $\pi$ :



**Question 17**

Find the area of this shape. Give your answer in terms of  $\pi$ :



**Question 18**

A cube has side lengths of 5cm.

Calculate its surface area.

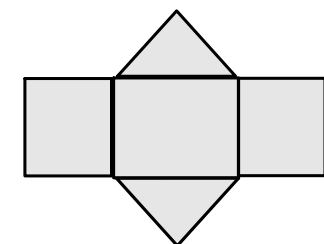
**Question 19**

A cuboid has height 30cm, width 6cm and length 20cm.

Calculate its surface area.

**Question 20**

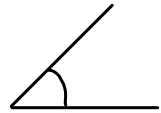
This is a net for which 3D shape?





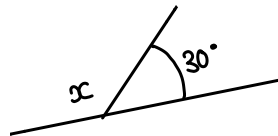
**Question 1**

What type of angle is this?



**Question 2**

Find the size of the missing angle:

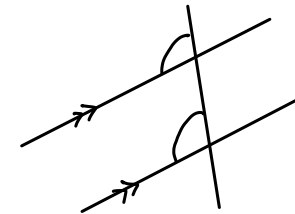


**Question 3**

What is the sum of the interior angles of a 10-sided polygon?

**Question 4**

These angles are equal. State the angle fact that this is showing.

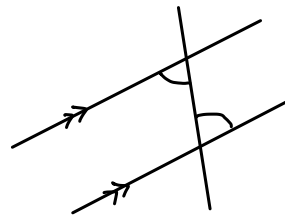


**Question 5**

A regular polygon has an exterior angle size of  $18^\circ$ . How many sides does the polygon have?

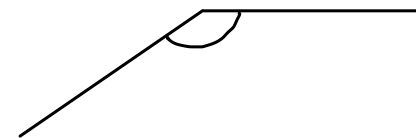
**Question 6**

These angles are equal. State the angle fact that this is showing.



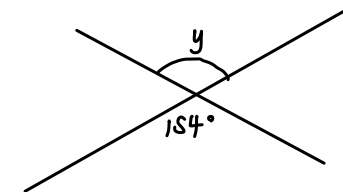
**Question 7**

What type of angle is this?



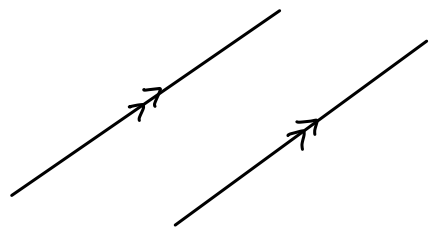
**Question 8**

Find the size of the missing angle:



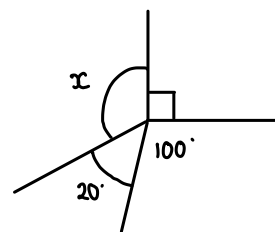
**Question 9**

Are these lines parallel or perpendicular?



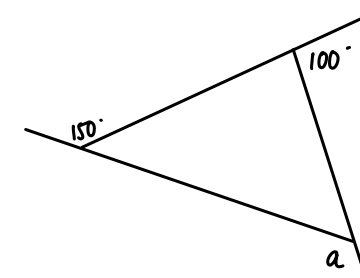
**Question 10**

Find the size of the missing angle:



**Question 11**

Find the size of the missing angle:

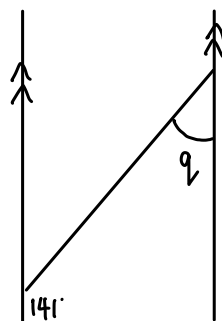


**Question 12**

What is the sum of the interior angles of a 12-sided polygon?

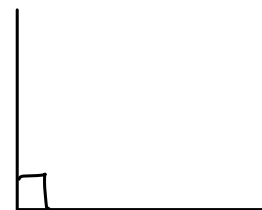
**Question 13**

Find the size of the missing angle:



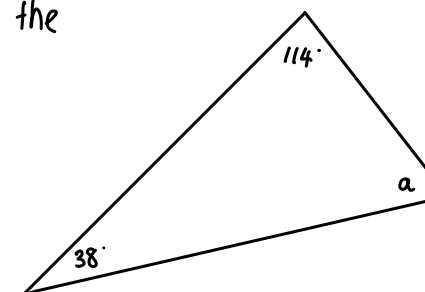
**Question 14**

What type of angle is this?



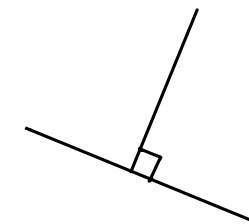
**Question 15**

Find the size of the missing angle:



**Question 16**

Are these lines parallel or perpendicular?



**Question 17**

Find the size of the missing angle:

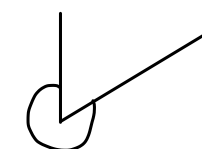


**Question 18**

A regular polygon has an exterior angle size of  $30^\circ$ . How many sides does the polygon have?

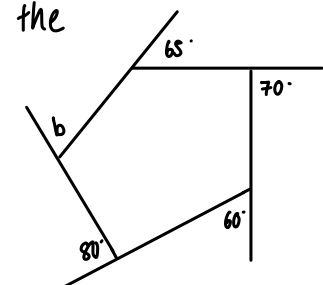
**Question 19**

What type of angle is this?



**Question 20**

Find the size of the missing angle:





# Year 9 – constructing in 2D/3D...



# Constructions & congruency

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Draw and measure angles
- Construct scale drawings
- Find locus of distance from points, lines, two lines
- Construct perpendiculars from points, lines, angles
- Identify congruence
- Identify congruent triangles

## Keywords

**Protractor:** piece of equipment used to measure and draw angles

**Locus:** set of points with a common property

**Equidistant:** the same distance

**Discorectangle:** (a stadium) – a rectangle with semi circles at either end

**Perpendicular:** lines that meet at  $90^\circ$

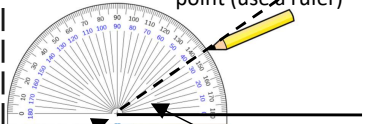
**Arc:** part of a curve

**Bisector:** a line that divides something into two equal parts

**Congruent:** the same shape and size

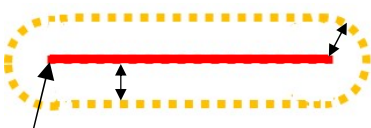
## Draw and measure angles

Make a mark at  $35^\circ$  with a pencil  
And join to the angle point (use a ruler)



Make sure the cross is at the end of the line (where you want the angle)

## Locus of a distance from a straight line



All points are equidistant (the same distance) from line

The ends of the line are fixed points

**Equipment needed**  
The line is straight so a ruler is used for the straight lines parallel to your original line

## Locus of a distance from two lines

**Also an angle bisector**  
This cuts the angle in half

From the angle vertex draw two arcs that cut the lines forming the angle  
Keep the compass the same size and use the new arcs as centres to draw intersecting arcs in the middle  
Join the vertex to the intersection

## Scale drawings

A picture of a car is drawn with a scale of 1:30

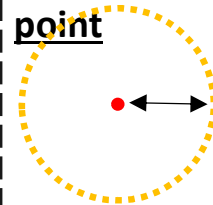
For every 1cm on my image is 30cm in real life

The car image is 10cm

Image : Real life  
 $1\text{cm} : 30\text{cm}$   
 $10\text{cm} : 300\text{cm}$

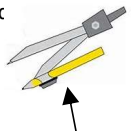


## Locus of a distance from a point



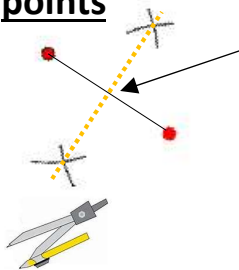
All points are equidistant (the same distance) from the fixed point in the mic

If the point is in the corner it can only make a quarter circle



**Equipment needed**  
The radius is the distance from the fixed point

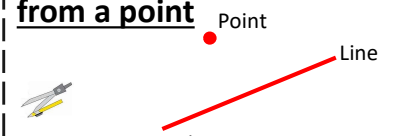
## Locus equidistant from two points



**Also a perpendicular bisector**  
Because if the points are joined, this new line intersects it at a  $90^\circ$

Join the intersections with a ruler.  
All points on this line are equidistant from both points

## Construct a perpendicular from a point



Use a compass and draw an arc that cuts the line. Use the point to place the compass. Keep the compass the same distance and now use your new points to make new intersecting arcs

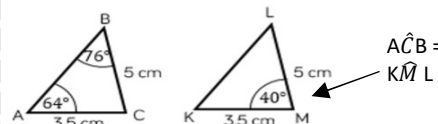
Connecting the arcs makes the bisector  
If P is a point on the line the steps are the same

## Congruent figures



Congruent figures are identical in size and shape – they can be reflections or rotations of each other

Congruent shapes are identical – all corresponding sides and angles are the same



Because all the angles are the same and  $AC=KM$   $BC=LM$  triangles ABC and KLM are congruent

## Congruent triangles

**Side-side-side**

All three sides on the triangle are the same size

**Angle-side-angle**

Two angles and the side connecting them are equal in two triangles

**Side-angle-side**

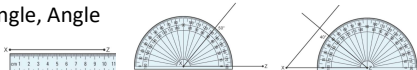
Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

**Right angle-hypotenuse-side**

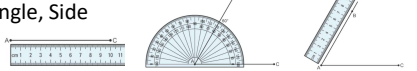
The triangles both have a right angle, the hypotenuse and one side are the same

## Constructing Triangles

Side, Angle, Angle



Side, Angle, Side

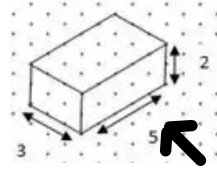


Side, Side, Side



**Question 1**

For this 3D solid, draw the plan:



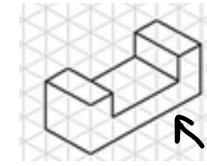
**Question 2**

Which congruency condition can be used to show that these triangles are congruent?



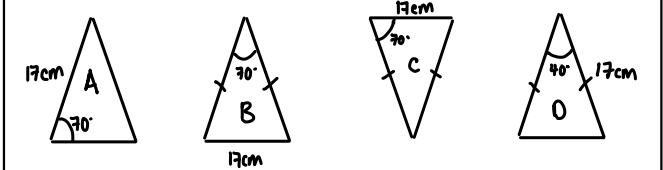
**Question 3**

For this 3D solid, draw the front elevation:



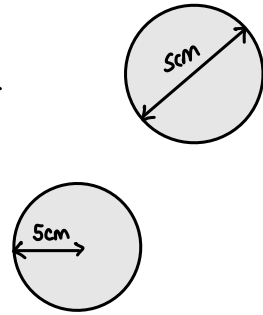
**Question 4**

Which two triangles are congruent?



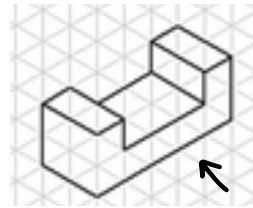
**Question 5**

Here are two circles. Are they congruent?



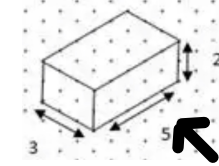
**Question 6**

For this 3D solid, draw the side elevation:



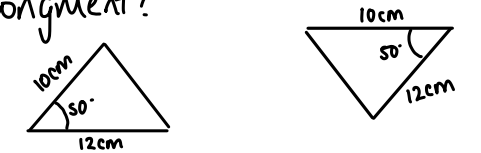
**Question 7**

For this 3D solid, draw the front elevation:



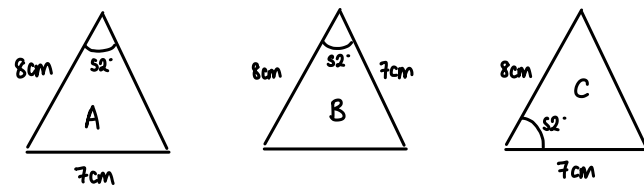
**Question 8**

Which congruency condition can be used to show that these triangles are congruent?



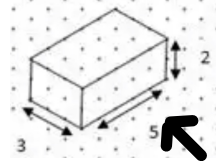
**Question 9**

Which two triangles are congruent?



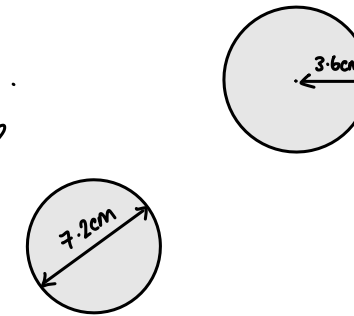
**Question 10**

For this 3D solid, draw the side elevation:



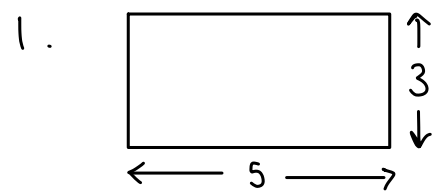
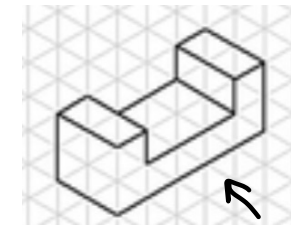
**Question 11**

Here are two circles. Are they congruent?

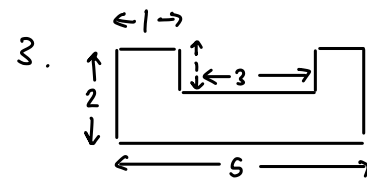


**Question 12**

For this 3D solid, draw the plan:

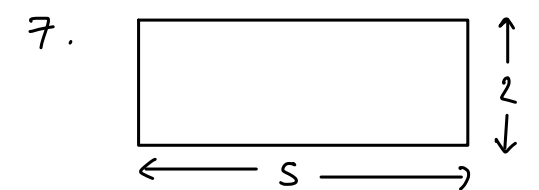
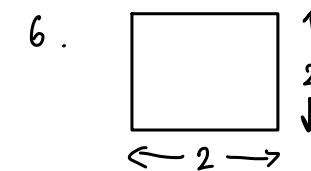


2. Side, side, side



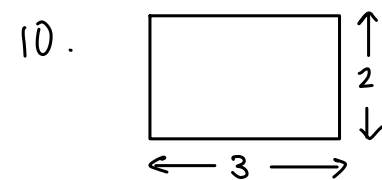
4. A and D

5. No



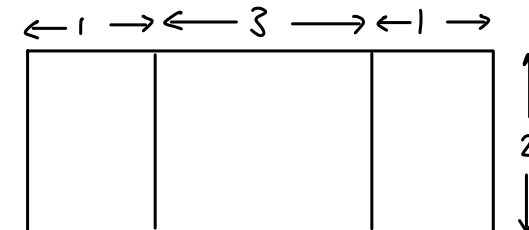
8. Side, angle, side

9. B and C



11. Yes

12.





# Maths & Money

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems with bills and bank statements
- Calculate simple interest
- Calculate compound interest
- Calculate wages and taxes
- Solve problems with exchange rates
- Solve unit pricing problems

## Keywords

- Credit:** money being placed into a bank account
- Debit:** money that leaves a bank account
- Balance:** the amount of money in a bank account
- Expense:** a cost/ outgoing.
- Deposit:** an initial payment (often a way of securing an item you will later pay for)
- Multiplier:** a number you are multiplying by. (Multiplier more than 1 = increasing, less than 1 = decreasing)
- Per Annum:** each year
- Currency:** the type of money a country uses.
- Unitary:** one – the cost of one.

## Bills and Bank Statements

**Bills** – tell you the amount items cost and can show how much money you need to pay.

Look for different units (Is it in pence or pounds)

Menu	Price
Milk	89p
Tea	£1.50

### Bank Statements

Bank statement can have negative balances if the money spent is higher than the money coming into the account

Date	Description	Credit	Debit	Balance
19 <sup>th</sup> Sept	Salary	£1500		£1500
19 <sup>th</sup> Sept	Mortgage		£600	£900
25 <sup>th</sup> Sept	Gift	£15		£915

## Simple Interest

For each year of investment the interest remains the same

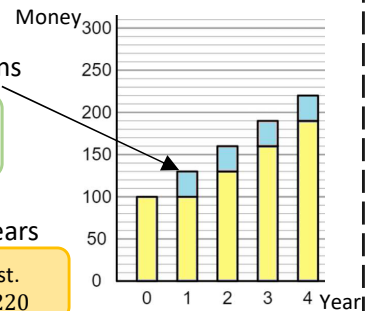
$$\frac{\text{Principal amount} \times \text{Interest Rate} \times \text{Years}}{100}$$

Principal amount is the amount invested in the account.

e.g. Invest £100 at 30% simple interest for 4 years

$$\frac{100 \times 30 \times 4}{100} = £120$$

This account earned £120 interest. At the end of year 4 they have £220



## Compound Interest

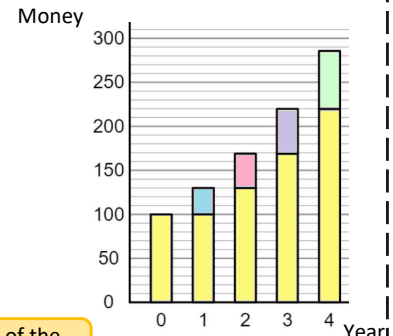
Interest is added to the current value of investment at the end of each year so the next year's interest is greater.

$$\text{Principal amount} \times \text{Multiplier}^{\text{Years}}$$

e.g. Invest £100 at 30% compound interest for 4 years

$$100 \times 1.3^4 = £285.61$$

This account has £285.61 in total at the end of the 4 years.



## Value Added Tax (VAT)

VAT is payable to the government by a business. In the UK VAT is 20% and added to items that are bought.

Essential items such as food do not include VAT.

## Wages and Taxes

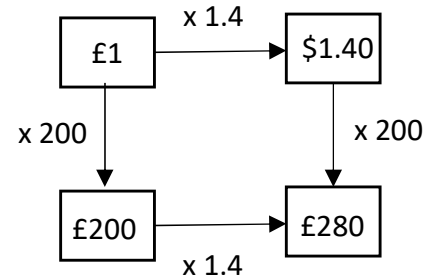
Salaries fall into tax brackets – which means they pay this much each month from their salary.

Taxable Income	Tax Rate
£12 501 to £50 000	20%
£50 001 to £150 000	40%
over £150 000	45%

Over time:

Time and a half – means 1.5 times their hourly rate  
Double – 2 times their hourly rate

## Exchange Rates



When making estimates it is also useful to use estimates to check if our solution is reasonable.

Use inverse operations to reverse the exchange process

### Common Currencies

United Kingdom	£	Pounds
United States of America	\$	Dollars
Europe	€	Euros

## Unit Pricing

4 Oranges £1	5 cupcakes £1.20
-----------------	---------------------

To calculate unit per cost you divide by the cost.

Cupcakes are the best value as one item has the cheapest value

$$4 = £1.00 \div 2 \quad 5 = £1.20 \div 5$$

$$2 = £0.50 \quad 1 = £0.25$$

$$1 = £0.25 \quad 1 = £0.20$$

Cost per Unit

There is a directly proportional relationship between the cost and number of units.

<p><b>Question 1</b> Calculate 50% of £40</p>	<p><b>Question 2</b> 144 trees were left standing out of 180 in an orchard. Calculate the percentage change.</p>	<p><b>Question 3</b> Work out the cost of an item after VAT is added Cost: £6,700</p>	<p><b>Question 4</b> A handbag costs £138 including VAT. How much does it cost without VAT?</p>
<p><b>Question 5</b> £2000 is deposited into an account that pays 5% simple interest. How much would be in the account after 1 year?</p>	<p><b>Question 6</b> \$1 = £0.55 How many pounds can you buy with 20 dollars?</p>	<p><b>Question 7</b> Calculate 65% of £20</p>	<p><b>Question 8</b> £2000 is deposited into an account that pays 5% <u>compound</u> interest. How much would be in the account after 5 years?</p>
<p><b>Question 9</b> Hannah is paid £11.70 an hour after tax is deducted. How much does she earn if she works for 10 hours?</p>	<p><b>Question 10</b> What is the multiplier for increasing an amount by 30%?</p>	<p><b>Question 11</b> John takes a loan of £12,000 over 3 years. He is charged compound interest at 4.2% per annum. How much interest will be charged over the 3 years?</p>	<p><b>Question 12</b> Hannah is paid £11.70 an hour after tax is deducted. She works 37.5 hours a week. How much does she earn in a week?</p>
<p><b>Question 13</b> Work out the cost of an item after VAT is added Cost: £899</p>	<p><b>Question 14</b> £2000 is deposited into an account that pays 5% simple interest. How much would be in the account after 2 years?</p>	<p><b>Question 15</b> A javelin thrower has a throw of 60m. Her next throw is 72m. What is her percentage increase?</p>	<p><b>Question 16</b> What is the multiplier for decreasing an amount by 30%?</p>
<p><b>Question 17</b> What does a negative account balance mean? (e.g. Account Balance -£215.70)</p>	<p><b>Question 18</b> A desk costs £180 after VAT has been added. What is the cost before VAT is added?</p>	<p><b>Question 19</b> £2000 is deposited into an account that pays 5% <u>compound</u> interest. How much would be in the account after 2 years?</p>	<p><b>Question 20</b> \$1 = £0.55 How many dollars can you buy with £33?</p>

# Year 10 – developing algebra...



## Simultaneous Equations

@whisto\_maths

### What do I need to be able to do?

By the end of this unit you should be able to:

- Determine whether (x,y) is a solution
- Solve by substituting a known variable
- Solve by substituting an expression
- Solve graphically
- Solve by subtracting/ adding equations
- Solve by adjusting equations
- Form and solve linear simultaneous equations

### Keywords

**Solution:** a value we can put in place of a variable that makes the equation true

**Variable:** a symbol for a number we don't know yet.

**Equation:** an equation says that two things are equal – it will have an equals sign =

**Substitute:** replace a variable with a numerical value

**LCM:** lowest common multiple (the first time the times table of two or more numbers match)

**Eliminate:** to remove

**Expression:** a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

**Coordinate:** a set of values that show an exact position.

**Intersection:** the point two lines cross or meet.

### Is (x, y) a solution?

x and y represent values that can be substituted into an equation

Does the coordinate (1,8) lie on the line  $y=3x+5$ ?

This coordinate represents  $x=1$  and  $y=8$   
 $y = 3x + 5$

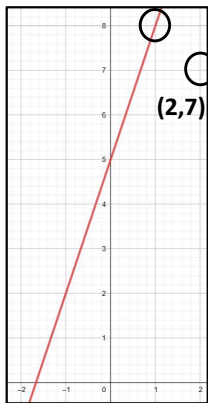
$$8 = 3(1) + 5$$

As the substitution makes the equation correct the coordinate (1,8) **IS** on the line  $y=3x+5$

Is (2,7) on the same line?

$$7 \neq 3(2) + 5$$

No 7 does NOT equal 6+5



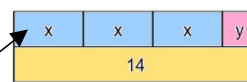
### Substituting known variables

Stephanie knows the point  $x = 4$  lies on that line. Find the value for  $y$ .

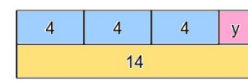
$$x = 4$$

A line has the equation  $3x + y = 14$

$$3x + y = 14$$



$$3(4) + y = 14$$



Two different variables, two solutions  
 $12 + y = 14$

$$-12 \quad -12$$

$$y = 2$$

### Substituting in an expression

Substitute  $2y$  in place of the  $x$  variable as they represent the same value

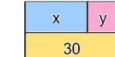
$$x = 2y$$



$$x + y = 30$$



$$x = 2y$$



$$x + y = 30$$

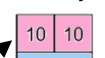


$$3y = 30$$



$$y = 10$$

$$x = 2y$$



$$x = 20$$

### Solve graphically

$$x + y = 6$$

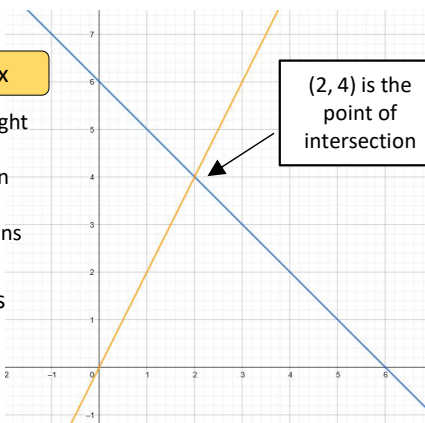
$$y = 2x$$

Linear equations are straight lines

The point of intersection provides the  $x$  and  $y$  solution for both equations

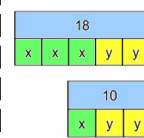
The solution that satisfies both equations is

$$x = 2 \text{ and } y = 4$$



(2, 4) is the point of intersection

### Solve by subtraction



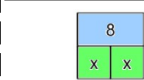
$$3x + 2y = 18$$

$$- \quad x + 2y = 10$$

$$2x = 8$$

$$\div 2 \quad \div 2$$

$$x = 4$$



$$x = 4$$

$$y = 3$$

$$x + 2y = 10$$

$$(4) + 2y = 10$$

$$-4 \quad -4$$

$$2y = 6$$

$$\div 2 \quad \div 2$$

$$y = 3$$

$$x + x + x + y + y = 18$$

$$x + y + y = 10$$

$$x + x + \cancel{y} + \cancel{y} = 18$$

$$\cancel{x} + \cancel{y} + \cancel{y} = 10$$

$$x + x = 8$$

$$x = 4$$

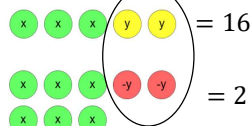
$$y = 3$$

### Solve by addition

Addition makes zero pairs

$$3x + 2y = 16$$

$$+ \quad 6x - 2y = 2$$



$$9x = 18$$

$$\div 9 \quad \div 9$$

$$x = 2$$

$$3x + 2y = 16$$

$$3(2) + 2(y) = 16$$

$$6 + 2y = 16$$

$$-6 \quad -6$$

$$2y = 10$$

$$y = 5$$



$$x = 2$$

$$y = 5$$

### Solve by adjusting one

No equivalent values

$$h + j = 12$$

$$2h + 2j = 29$$

$$12$$

$$29$$

$$24$$

$$29$$

$$2h + 2j = 24$$

$$2h + 2j = 29$$

By proportionally adjusting one of the equations – now solve the simultaneous equations choosing an addition or subtraction method

### Solve by adjusting both

$$2x + 3y = 39$$

$$5x - 2y = -7$$

$$x + x + y + y + y = 39$$

$$x + x + x + \cancel{y} + \cancel{y} = -7$$

Use LCM to make equivalent  $x$  OR  $y$  values.

Because of the negative values using zero pairs and  $y$  values is chosen choice

$$4x + 6y = 78$$

$$15x - 6y = -21$$

$$x + x + x + y + y + y = 78$$

$$x + x + x + x + x + \cancel{y} + \cancel{y} + \cancel{y} = -21$$

Now solve by addition

Addition makes zero pairs

<b>Question 1</b> make $x$ the subject of $x + b = c$	<b>Question 2</b> Solve simultaneously: $2x - y = 1$ $3x + y = 9$	<b>Question 3</b> make $x$ the subject of $3x = d$	<b>Question 4</b> Solve simultaneously: $5x + y = 17$ $3x + y = 11$
<b>Question 5</b> Solve simultaneously: $2x + 3y = 9$ $2x + y = 7$	<b>Question 6</b> make $x$ the subject of $ax + b = c$	<b>Question 7</b> Solve simultaneously: $4x - 3y = 14$ $2x + 3y = 16$	<b>Question 8</b> make $x$ the subject of $a = x^2 + 8$
<b>Question 9</b> Solve simultaneously: $3x + y = 10$ $5x + 2y = 17$	<b>Question 10</b> make $x$ the subject of $c = \frac{x}{a}$	<b>Question 11</b> Solve simultaneously: $4x - 2y = 8$ $3x + 6y = 21$	<b>Question 12</b> make $x$ the subject of $x - c = f$
<b>Question 13</b> make $x$ the subject of $t = \sqrt{x} + 8$	<b>Question 14</b> Solve simultaneously: $3x + 7y = 2b$ $5x + 2y = 24$	<b>Question 15</b> make $x$ the subject of $\frac{x+3}{2} = a$	<b>Question 16</b> Solve simultaneously: $3x - 2y = 7$ $5x + 3y = 37$
<b>Question 17</b> Solve simultaneously: $5x + 3y = 41$ $2x + 3y = 20$	<b>Question 18</b> make $x$ the subject of $2x + 4 = f$	<b>Question 19</b> Solve simultaneously: $5x + y = 11$ $3x - y = 9$	<b>Question 20</b> make $y$ the subject of $x = \frac{y+3}{9}$

# Pythagoras' theorem

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Use square and cube roots
- Identify the hypotenuse
- Calculate the hypotenuse
- Find a missing side in a Right angled triangle
- Use Pythagoras' theorem on axes
- Explore proofs of Pythagoras' theorem.

## Keywords

**Square number:** the output of a number multiplied by itself

**Square root:** a value that can be multiplied by itself to give a square number

**Hypotenuse:** the largest side on a right angled triangle. Always opposite the right angle.

**Opposite:** the side opposite the angle of interest

**Adjacent:** the side next to the angle of interest

## Squares and square roots

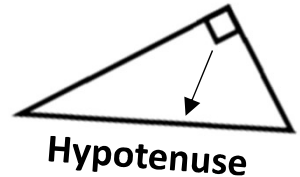
This can also be written as  $6^2$

$\sqrt{\quad}$  is the square root symbol  
e.g.  $\sqrt{64} = 8$   
Because  $8 \times 8 = 64$

1 × 1	2 × 2	3 × 3	4 × 4	5 × 5	6 × 6	7 × 7	8 × 8	9 × 9	10 × 10
1	4	9	16	25	36	49	64	81	100

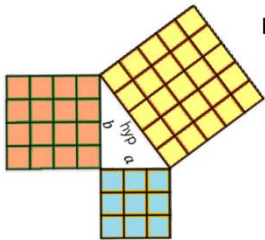
Square numbers

## Identify the hypotenuse



The hypotenuse is always the longest side on a triangle because it is opposite the biggest angle.

## Determine if a triangle is right-angled.



If a triangle is right-angled, the sum of the squares of the shorter sides will equal the square of the hypotenuse.

$$a^2 + b^2 = \text{hypotenuse}^2$$

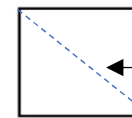
e.g.  $a^2 + b^2 = \text{hypotenuse}^2$

$$3^2 + 4^2 = 5^2$$

$$9 + 16 = 25$$

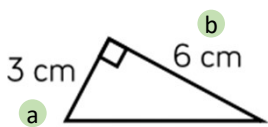
Substituting the numbers into the theorem shows that this is a right-angled triangle

$a = 3$   $b = 4$   $c = 5$



Polygons can still have a hypotenuse if it is split up into triangles and opposite a right angle

## Calculate the hypotenuse



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

1. Substitute in the values for a and b

$$3^2 + 6^2 = \text{hypotenuse}^2$$

$$9 + 36 = \text{hypotenuse}^2$$

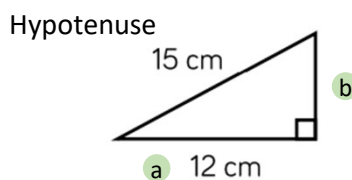
$$45 = \text{hypotenuse}^2$$

2. To find the hypotenuse square root the sum of the squares of the shorter sides.

$$\sqrt{45} = \text{hypotenuse}$$

$$6.71 \text{ cm} = \text{hypotenuse}$$

## Calculate missing sides



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

$$12^2 + b^2 = 15^2$$

1. Substitute in the values you are given

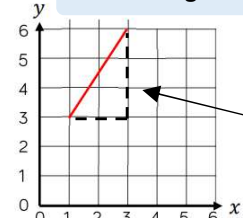
$$144 + b^2 = 225$$

Rearrange the equation by subtracting the shorter square from the hypotenuse squared

$$\text{Square root to find the length of the side} \left\{ \begin{array}{l} b^2 = 111 \\ b = \sqrt{111} = 10.54 \text{ cm} \end{array} \right.$$

## Pythagoras' theorem on a coordinate axis

Find the length of the line segment



The segment can be made into a right-angled triangle by adding the sides on the diagram.

The line segment is the hypotenuse

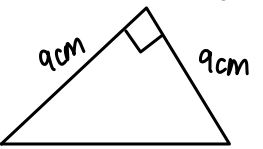
$$a^2 + b^2 = \text{hypotenuse}^2$$

The lengths of a and b are the sides of the triangle.

Be careful to check the scale on the axes

**Question 1**  
True or false:  
 $2 \cdot 1^2 = \sqrt{4 \cdot 2}$

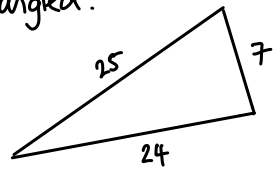
**Question 2**  
Calculate the length of the hypotenuse to 1 decimal place.



**Question 3**  
True or false:  
The hypotenuse of a right-angled triangle is always opposite the right angle.

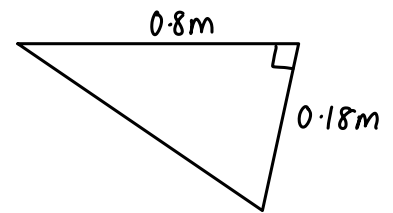
**Question 4**  
Calculate the distance between the coordinates:  
(0,0) (6,8)

**Question 5**  
Use Pythagoras' theorem to determine whether this triangle is right-angled.



**Question 6**  
Calculate the distance between the coordinates:  
(8,6) (7,5)

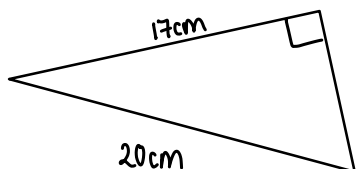
**Question 7**  
Calculate the length of the hypotenuse to 1 decimal place.



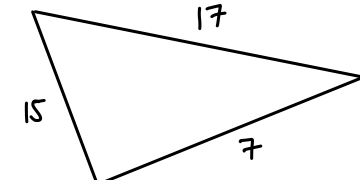
**Question 8**  
True or false:  
 $\sqrt{0.09} = 0.3$

**Question 9**  
Calculate the distance between the coordinates:  
(1,3) (4,7)

**Question 10**  
Calculate the unknown length to 1 decimal place.

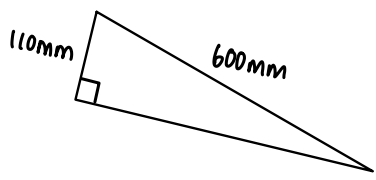


**Question 11**  
Use Pythagoras' theorem to determine whether this triangle is right-angled.

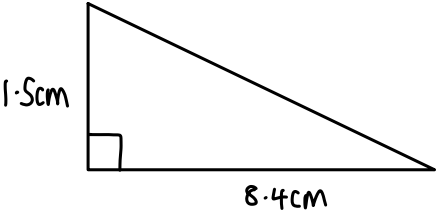


**Question 12**  
Calculate the distance between the coordinates:  
(10,1) (5,3)

**Question 13**  
Calculate the unknown length to 1 decimal place.



**Question 14**  
Calculate the length of the hypotenuse to 1 decimal place.



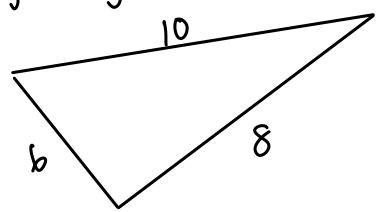
**Question 15**  
Calculate the distance between the coordinates:  
(-5,5) (-1,3)

**Question 16**  
True or false:  
The hypotenuse is the largest side of any triangle.

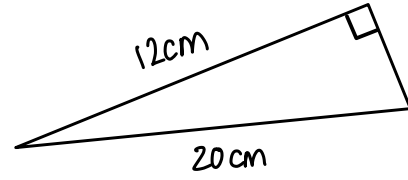
**Question 17**  
True or false:  
Only right-angled triangles have a hypotenuse.

**Question 18**  
Calculate the distance between the coordinates:  
(-3,-2) (2,1)

**Question 19**  
Use Pythagoras' theorem to determine whether this triangle is right-angled.



**Question 20**  
Calculate the unknown length to 1 decimal place.





# Enlargement & Similarity

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Recognise enlargement and similarity
- Enlarge a shape by a positive SF
- Enlarge a shape from a point
- Enlarge a shape by a fractional SF
- Work out missing sides and angles in a pair of similar shapes.

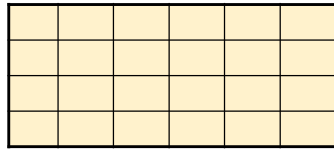
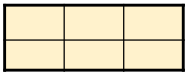
## Keywords

- Similar Shapes:** shapes of different sizes that have corresponding sides in equal proportion and identical corresponding angles.
- Scale Factor:** the multiple describing how much a shape has been enlarged
- Enlarge:** to change the size of a shape (enlargement is not always making a shape bigger)
- Corresponding:** objects (or sides) that appear in the same place in two similar situations.
- Image:** the picture or visual representation

## Recognise enlargement & similarity

Shapes are similar if all pairs of corresponding sides are in the same ratio

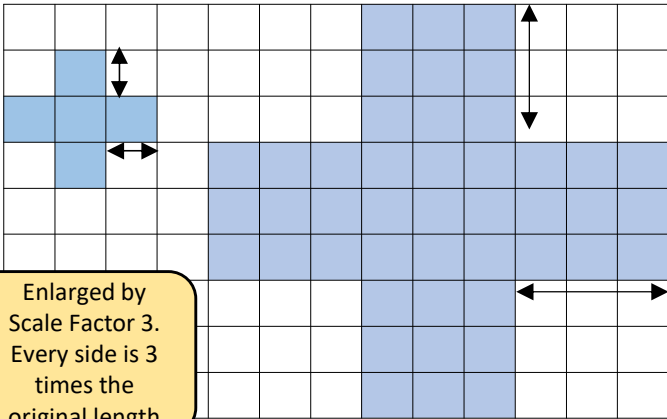
These shapes are similar because all sides are increased by the same ratio



Enlargements are similar shapes with a ratio other than 1

## Enlarge by a positive scale factor

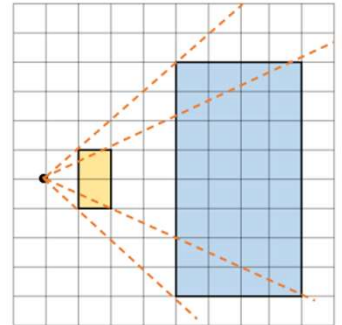
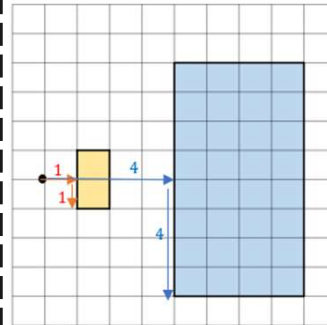
With a scale factor larger than 1 it makes the shape **bigger**



## Enlarge a shape from a point

Scaled distances method

Rays method



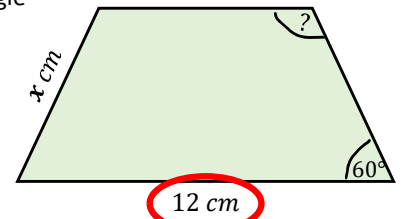
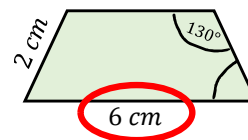
Scale the distance between the point of enlargement and each corresponding vertices

Multiply the distance from the centre of corresponding vertices by the scale factor along the ray

## Calculations in similar shapes

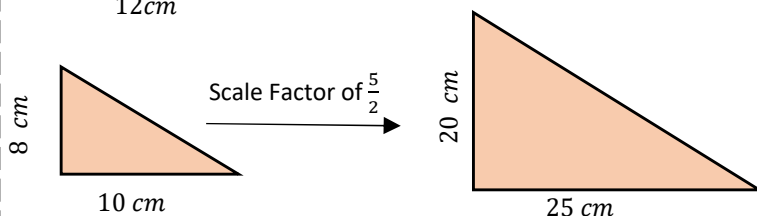
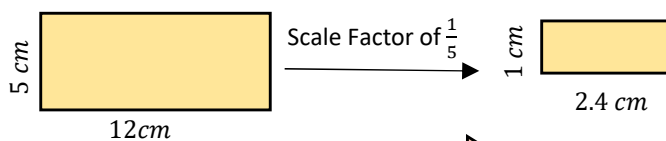
Don't forget that properties of shapes don't change with enlargements or in similar shapes

The two trapezium are similar find the missing side and angle



## Positive fractional scale factor

With a scale factor between 0 and 1 it makes the shape **smaller**



**Corresponding sides identify the scale factor**

$$\frac{12}{6} = 2 \quad \text{Scale Factor} = 2$$

**Calculate the missing side**

Length (corresponding side) x scale factor

$$2\text{ cm} \times 2 = x = 4\text{ cm}$$

Enlargement does not change angle size

**Calculate the missing angle**

Corresponding angles remain the same

130°

# Rotation & Translation

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Identify the order of rotational symmetry
- Rotate a shape about a point on the shape
- Rotate a shape about a point not on a shape
- Translate by a given vector
- Compare rotations and reflections

## Keywords

**Rotate:** a rotation is a circular movement.

**Symmetry:** when two or more parts are identical after a transformation.

**Regular:** a regular shape has angles and sides of equal lengths.

**Invariant:** a point that does not move after a transformation.

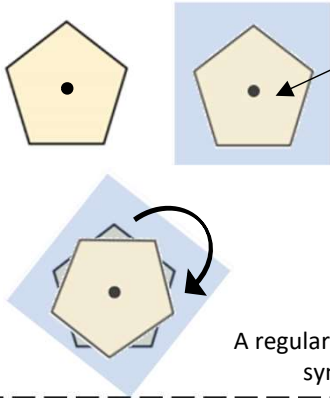
**Vertex:** a point two edges meet.

**Horizontal:** from side to side

**Vertical:** from up to down

## Rotational Symmetry

Tracing paper helps check rotational symmetry.



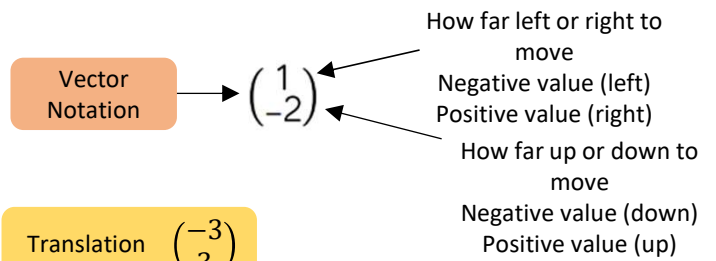
1. Trace your shape (mark the centre point)

2. Rotate your tracing paper on top of the original through 360°

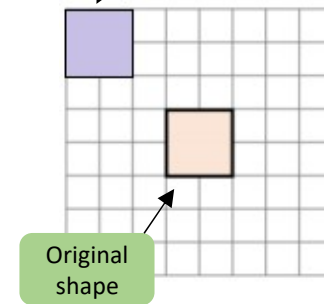
3. Count the times it fits back into itself

A regular pentagon has rotational symmetry of order 5

## Translation and vector notation



Translation  $\begin{pmatrix} -3 \\ 3 \end{pmatrix}$



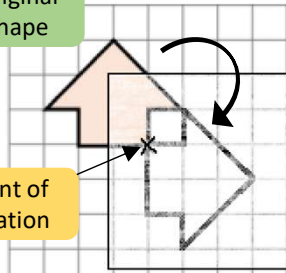
Every vertex has been translated by the same amount

$$\begin{pmatrix} -3 \\ 3 \end{pmatrix}$$

The image has been moved 3 squares to the left and 3 squares up

## Rotate from a point (in a shape)

Original shape



Point of rotation

Image: 90° clockwise

1. Trace the original shape (mark the point of rotation)

2. Keep the point in the same place and turn the tracing paper

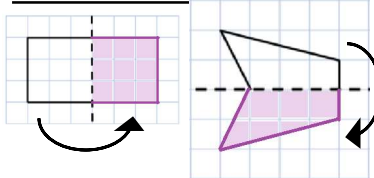
3. Draw the new shape



Clockwise

Anti-Clockwise

## Compare rotations and reflections



**R**

Reflections are a mirror image of the original shape.

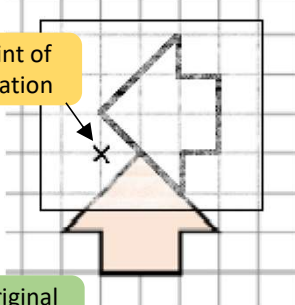
Information needed to perform a reflection:

- Line of reflection (Mirror line)

## Rotate from a point (outside a shape)

Image: 90° anti-clockwise

Point of rotation



Original shape

1. Trace the original shape (mark the point of rotation)

2. Keep the point in the same place and turn the tracing paper

3. Draw the new shape

Rotations are the movement of a shape in a circular motion

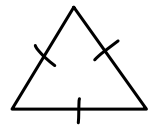
Information needed to perform a rotation:

- Point of rotation
- Direction of rotation
- Degrees of rotation

Transformations

**Question 1**

State the order of rotational symmetry

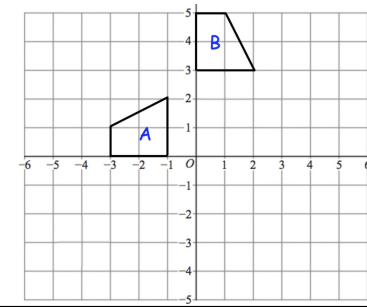


**Question 2**

What is the key information you must include when describing a rotation?

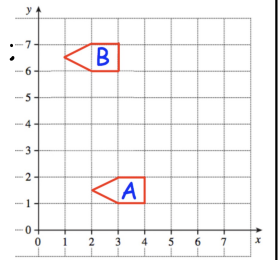
**Question 3**

Describe the transformation: from A to B



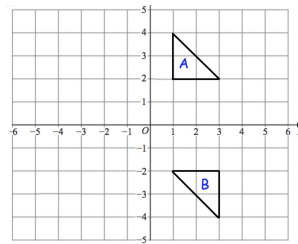
**Question 4**

Describe the transformation: from A to B



**Question 5**

Describe the transformation: from A to B

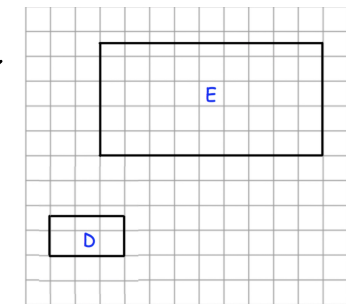


**Question 6**

What is the key information you must include when describing a translation?

**Question 7**

What is the scale factor of the enlargement?

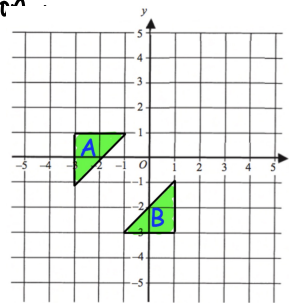


**Question 8**

What is the key information you must include when describing an enlargement?

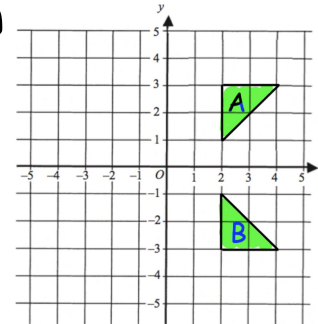
**Question 9**

Describe the transformation: from A to B



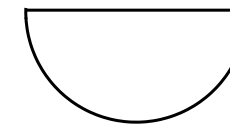
**Question 10**

Describe the transformation: from A to B



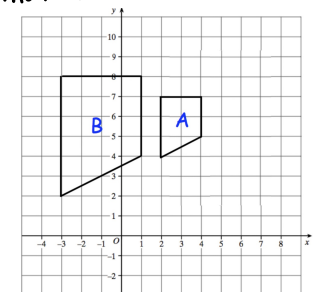
**Question 11**

State the order of rotational symmetry



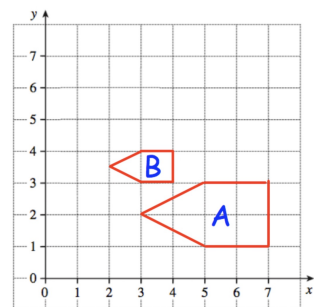
**Question 12**

Describe the transformation: from A to B



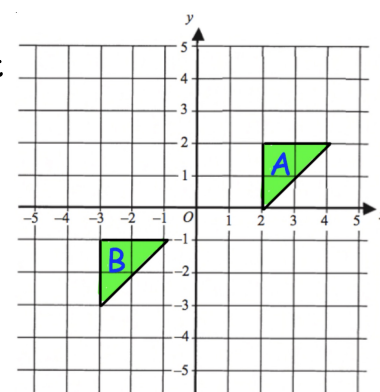
**Question 13**

Describe the transformation: from B to A



**Question 14**

Describe the transformation: from A to B

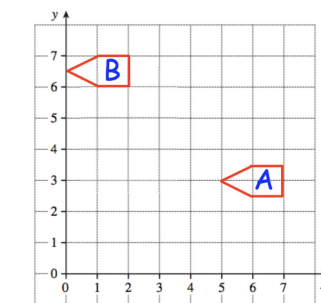


**Question 15**

What is the key information you must include when describing a reflection?

**Question 16**

Describe the transformation: from A to B



- Answers: 1. 3    2. Angle, direction, centre    3. Rotation 90° clockwise through (1, 1)    4. Translation  $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$     5. Rotation, 180° through (2, 0)    6. Column vector    7. sf 3    8. Scale factor, centre of enlargement
9. Reflection in  $y=x$     10. Reflection in x-axis,  $y=0$     11. 1    12. Enlargement, sf 2 through (7, 6)    13. Enlargement, sf 2 through (1, 5)    14. Translation  $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$     15. Equation of line of reflection    16. Translation  $\begin{pmatrix} -5 \\ 3.5 \end{pmatrix}$

# Year 9 – similarity...

# Trigonometry

@whisto\_maths

## What do I need to be able to do?

At the end of this unit you should be able to:

- Work fluently with hypotenuse, opposite and adjacent sides
- Use the tan, sine and cosine ratio to find missing side lengths
- Use the tan, sine and cosine ratio to find missing angles
- Calculate sides using Pythagoras' Theorem

## Keywords

**Enlarge:** to make a shape bigger (or smaller) by a given multiplier (scale factor)

**Scale Factor:** the multiplier of enlargement

**Constant:** a value that remains the same

**Cosine ratio:** the ratio of the length of the adjacent side to that of the hypotenuse. The sine of the complement.

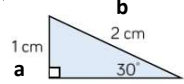
**Sine ratio:** the ratio of the length of the opposite side to that of the hypotenuse.

**Tangent ratio:** the ratio of the length of the opposite side to that of the adjacent side.

**Inverse:** function that has the opposite effect.

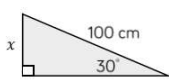
**Hypotenuse:** longest side of a right-angled triangle. It is the side opposite the right-angle.

## Ratio in right-angled triangles



$$a : b = 1 : 2$$

$$\frac{a}{b} = \frac{1}{2}$$



$$a : b = x : 50 = 100$$

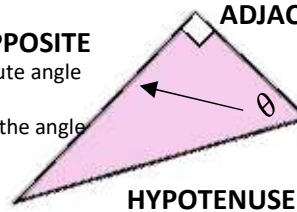
When the angle is the same the ratio of sides a and b will also remain the same



$$a : b = 0.07 : x = 0.07 : 0.14$$

## Hypotenuse, adjacent and opposite

Always opposite an acute angle  
Useful to label second  
Position depend upon the angle in use for the question



Next to the angle in question  
Often labelled last

Always the longest side  
Always opposite the right angle  
Useful to label this first

## Tangent ratio: side lengths

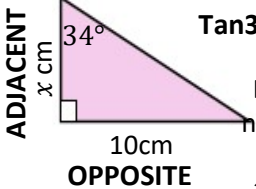
$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Substitute the values into the tangent formula

$$\tan 34 = \frac{10}{x}$$

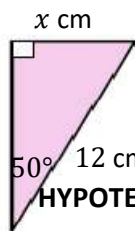
Equations might need rearranging to solve  
 $x \times \tan 34 = 10$

$$x = \frac{10}{\tan 34} = 14.8 \text{ cm}$$



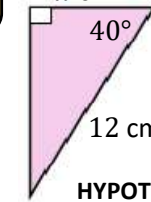
## Sin and Cos ratio: side lengths

OPPOSITE



$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$$

ADJACENT



$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse side}}$$

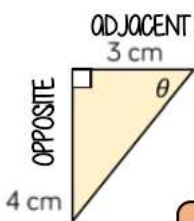
Substitute the values into the ratio formula

**NOTE**  
The Sin(x) ratio is the same as the Cos(90-x) ratio

Equations might need rearranging to solve

## Sin, Cos, Tan: Angles

### Inverse trigonometric functions



Label your triangle and choose your trigonometric ratio  
Substitute values into the ratio formula

$$\theta = \tan^{-1} \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\theta = \sin^{-1} \frac{\text{opposite side}}{\text{hypotenuse side}}$$

$$\theta = \cos^{-1} \frac{\text{adjacent side}}{\text{hypotenuse side}}$$

$$\tan \theta = \frac{3}{4}$$

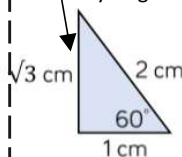
$$\theta = \tan^{-1} \frac{3}{4}$$

$$\theta = 36.9^\circ$$

## Key angles

Because trig ratios remain the same for similar shapes you can generalise from the following statements.

This side could be calculated using Pythagoras



$$\tan 30 = \frac{1}{\sqrt{3}}$$

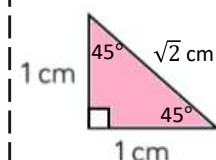
$$\cos 30 = \frac{\sqrt{3}}{2}$$

$$\sin 30 = \frac{1}{2}$$

$$\tan 60 = \sqrt{3}$$

$$\cos 60 = \frac{1}{2}$$

$$\sin 60 = \frac{\sqrt{3}}{2}$$



$$\tan 45 = 1$$

$$\cos 45 = \frac{1}{\sqrt{2}}$$

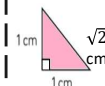
$$\sin 45 = \frac{1}{\sqrt{2}}$$

## Key angles 0° and 90°

$$\tan 0 = 0$$

~~$$\tan 90$$~~

This value cannot be defined – it is impossible as you cannot have two 90° angles in a triangle



$$\sin 0 = 0$$

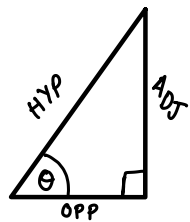
$$\sin 90 = 1$$

$$\cos 0 = 1$$

$$\cos 90 = 0$$

**Question 1**

True or false:  
This triangle is labelled correctly

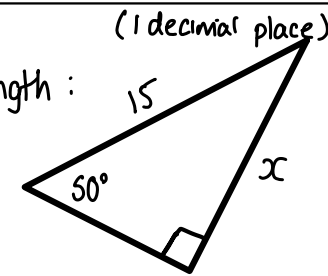


**Question 2**

What is the exact value of  $\sin 30^\circ$ ?

**Question 3**

Calculate the missing side length:

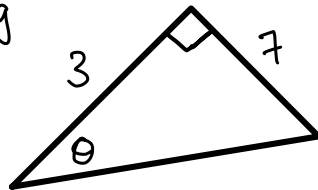


**Question 4**

What is the exact value of  $\sin 90^\circ$ ?

**Question 5**

Calculate the missing angle:



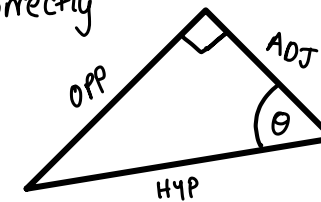
(1 decimal place)

**Question 6**

What is the exact value of  $\cos 30^\circ$ ?

**Question 7**

True or false:  
This triangle is labelled correctly



**Question 8**

What is the exact value of  $\sin 60^\circ$ ?

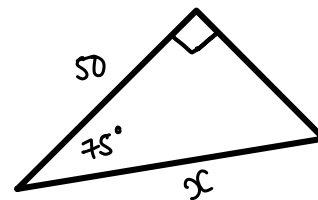
**Question 9**

What is the exact value of  $\cos 45^\circ$ ?

**Question 10**

Calculate the missing side length:

(1 decimal place)



**Question 11**

What is the exact value of  $\tan 30^\circ$ ?

**Question 12**

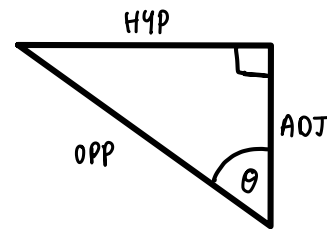
What is the exact value of  $\cos 0^\circ$ ?

**Question 13**

What is the exact value of  $\tan 45^\circ$ ?

**Question 14**

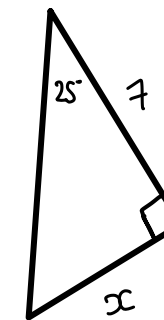
True or false:  
This triangle is labelled correctly



**Question 15**

Calculate the missing side length:

(1 decimal place)



**Question 16**

What is the exact value of  $\cos 60^\circ$ ?

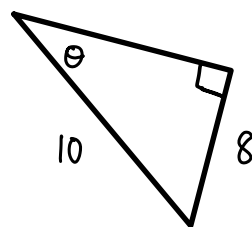
**Question 17**

What is the exact value of  $\sin 0^\circ$ ?

**Question 18**

Calculate the missing angle

(1 decimal place)



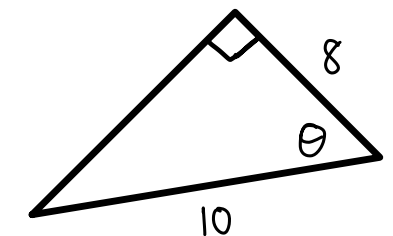
**Question 19**

What is the exact value of  $\tan 60^\circ$ ?

**Question 20**

Calculate the missing angle

(1 decimal place)





# Year 9 - Reasoning with data...



## Measures of location

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### What do I need to be able to do?

By the end of this unit you should be able to:

- Understand and use mean, median and mode
- Choose the most appropriate average
- Identify outliers
- Compare distributions using averages and range

### Keywords

**Spread:** the distance/ how spread out/ variation of data

**Average:** a measure of central tendency – or the typical value of all the data together

**Total:** all the data added together

**Frequency:** the number of times the data values occur

**Represent:** something that shows the value of another

**Outlier:** a value that stands apart from the data set

**Consistent:** a set of data that is similar and doesn't change very much

### Mean, Median, Mode

#### The Mean

A measure of average to find the central tendency... a typical value that represents the data

24, 8, 4, 11, 8,

Find the sum of the data (add the values)  $55$

Divide the overall total by how many pieces of data you have  $55 \div 5$

**Mean = 11**

#### The Median

The value in the centre (in the middle) of the data

24, 8, 4, 11, 8,

Put the data in order **4, 8, 8, 11, 24**

Find the value in the middle **4, 8, 8, 11, 24**

**Median = 8**

**NOTE:** If there is no single middle value find the mean of the two numbers left

#### The Mode (The modal value)

This is the number OR the item that occurs the most (it does not have to be numerical)

24, 8, 4, 11, 8,

This can still be easier if the data is ordered first

**Mode = 8**

**4, 8, 8, 11, 24**

### Choosing the appropriate average

The average should be a representative of the data set – so it should be compared to the set as a whole - to check if it is an appropriate average

Here are the weekly wages of a small firm

£240 £240 £240 £240 £240  
£260 £260 £300 £350 £700

Which average best represents the weekly wage?

The Mean = £307

The Median = £250

The Mode = £240

**Put the data back into context**

Mean/Median – too high (most of this company earn £240)  
Mode is the best average that represents this wage

It is likely that the salaries above £240 are more senior staff members – their salary doesn't represent the average weekly wage of the majority of employers

### Identify outliers

Outliers are values that stand well apart from the rest of the data

Outliers can have a big impact on range and mean.

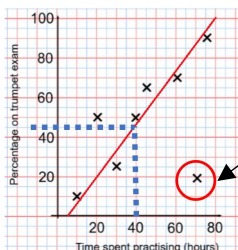
They have less impact on the median

Height in cm  
152 150 142 158 (182) 151 153 149 156 160 151 144

Sometimes it is best to not use an outlier in calculations

Where an outlier is identified try to give it some context.

This is likely to be a taller member of the group. Could the be an older student or a teacher?



Outliers can also be identified graphically e.g. on scatter graphs

### Comparing distributions

Comparisons should include a statement of average and central tendency, as well as a statement about spread and consistency.

Here are the number of runs scored last month by Lucy and James in cricket matches

**Lucy:** 45, 32, 37, 41, 48, 35

**James:** 60, 90, 41, 23, 14, 23

James has two extreme values that have a big impact on the range

Lucy

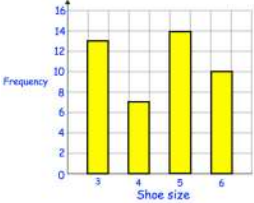
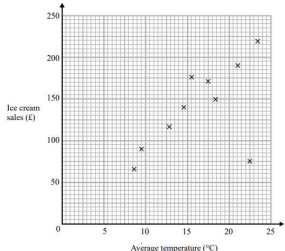
**Mean:** 39.6 (1.d.p), **Median:** 38. **Mode:** no mode, **Range:** 16

James

**Mean:** 41.8 (1.d.p), **Median:** 32, **Mode:** 23, **Range:** 76

"James is less consistent than Lucy because his scores have a greater range. Lucy performed better on average because her scores have a similar mean and a higher median"



<p>Question 1</p> <p>Work out the mode for the following data:</p> <p>5, 6, 6, 7, 8, 10</p>	<p>Question 2</p> <p>Work out the mode for the following data:</p> <p>5, 5, 7, 7, 7, 8, 8, 9</p>	<p>Question 3</p> <p>Work out the mode for the following data:</p> <p>5, 7, 3, 1, 8, 9, 10, 2</p>	<p>Question 4</p> <p>Work out the mode for the following data:</p> <p>-2, -1, 5, 8, -2, 2, -1, 9, -1, 1, 2, -1</p>	<p>Question 5</p>  <p>What is the modal shoe size?</p>									
<p>Question 6</p> <p>Work out the median for the following data:</p> <p>5, 1, 4, 6, 8</p>	<p>Question 7</p> <p>Work out the median for the following data:</p> <p>6.2, 6.8, 6.6, 7.2, 6.4, 7.4, 5.8</p>	<p>Question 8</p> <p>Work out the median for the following data:</p> <p>9, 8, 6, 6, 6, 7, 1, 2, 6, 8</p>	<p>Question 9</p> <p>Work out the median for the following data:</p> <p>-4, 5, -7, -1, 2, 0, 9</p>	<p>Question 10</p> <p>The height of some footballers are listed below:</p> <p>1.81m, 1.78m, 1.88m, 1.79m, 1.86m, 1.85m, 1.78m, 1.93m</p> <p>Work out the median height.</p>									
<p>Question 11</p> <p>Find the mean of the following data:</p> <p>4, 9, 7, 10, 5</p>	<p>Question 12</p> <p>Find the mean of the following data:</p> <p>3, 2, 1, 3, 2, 2, 1, 3, 1, 2, 3, 2, 1</p>	<p>Question 13</p> <p>Find the mean of the following data:</p> <p>9, -3, -6, 5, 0</p>	<p>Question 14</p> <p>Find the mean of the following data:</p> <p>1.4, 2.8, 2.4, 2.5, 2.8, 3.1, 1.1</p>	<p>Question 15</p> <p>Five houses on a street are sold in 2016. They sell for</p> <p>£175,000 £184,000 £150,000 £201,000 £191,000</p> <p>Calculate the mean price.</p>									
<p>Question 16</p> <p>Find the missing word:</p> <p>The mean, median and mode are known as measures of</p> <p>-----</p>	<p>Question 17</p> <p>Identify the outlier:</p> 	<p>Question 18</p> <p>Which average does outliers affect most?</p>	<p>Question 19</p> <p>Compare the distances travelled by snails and slugs over the duration of ten minutes.</p> <table border="1" data-bbox="1339 1329 1711 1394"> <thead> <tr> <th></th> <th>Median</th> <th>Interquartile range</th> </tr> </thead> <tbody> <tr> <td>Snails</td> <td>7.1 cm</td> <td>3.1 cm</td> </tr> <tr> <td>Slugs</td> <td>9.7 cm</td> <td>4.5 cm</td> </tr> </tbody> </table>		Median	Interquartile range	Snails	7.1 cm	3.1 cm	Slugs	9.7 cm	4.5 cm	<p>Question 20</p> <p>A teacher asks a group of students how much pocket money they receive each week. They respond:</p> <p>£5 £8 £4 £50 £6 £8 £7.50 £10 £8 £7</p> <p>Which average, the median or the mean, is most suitable for this data?</p>
	Median	Interquartile range											
Snails	7.1 cm	3.1 cm											
Slugs	9.7 cm	4.5 cm											

# Numbers

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## What do I need to be able to do?

By the end of this unit you should be able to:

- Identify integers, real and rational numbers
- Work with directed number
- Solve problems with number
- Find HCF/ LCM
- Add/ Subtract fractions
- Multiply/ Divide fractions
- Write numbers in standard form

## Keywords

- Integer:** a whole number that is positive or negative  
**Rational:** a number that can be made by dividing two integers  
**Irrational:** a number that cannot be made by dividing two integers  
**Inverse operation:** the operation that reverses the action  
**Quotient:** the result of a division  
**Product:** the result of a multiplication.  
**Multiples:** found by multiplying any number by positive integers  
**Factor:** integers that multiply together to get another number

## Integers, real and rational numbers

**Rational** – root word: ratio

**Real numbers:**  $\frac{2}{3}$  stems from 2:1 ( $\frac{2}{3}$  of the whole)

**Irrational numbers:**  $\sqrt{2}$  the solution is a decimal that never ends and does not repeat.

The square root of a negative is not a real number and cannot be found.

## HCF/LCM

1 is a common factor of all numbers

Common factors are factors two or more numbers share

### HCF – Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30 **HCF = 6**

### LCM – Lowest common multiple

LCM of 9 and 12

LCM = 36

9: 9, 18, 27, 36, 45, 54

12: 12, 24, 36, 48, 60

The first time their multiples match

## Standard form

Any number between 1 and less than 10  $\rightarrow A \times 10^n$  Any integer

$6 \times 10^5 + 8 \times 10^5$

$= 600000 + 800000$

$= 1400000$

$= 1.4 \times 10^6$

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

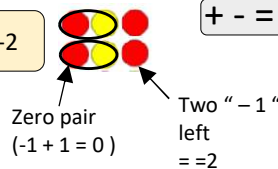
$1.5 \div 0.3 \times 10^5 \div 10^3$

$= 5 \times 10^2$

## Directed number

### Addition

$2 + -4 = -2$



Generalisation  $+ - = -$

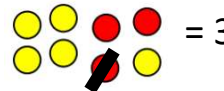
### Subtraction

$2 - -1 = 3$

Representation for calculation

$2 - -1 = 3$

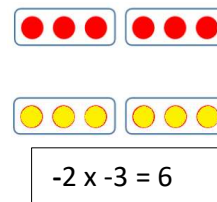
Take away (Start with the representation of 2)



"Subtract" – means take away or remove

Generalisation  $- - = +$

### Multiplication



Red dot = -1  
Yellow dot = 1

The act of making counters into their negative is turning them over

Divisions are the inverse operations



$a = 5$

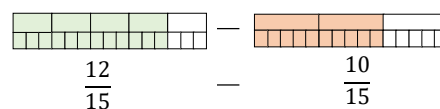
$b = -4$

Brackets around negative substitutions helps remove calculation errors

$2a - b = 2 \times 5 - (-4) = 10 + 4 = 14$

## Addition/ Subtraction of fractions

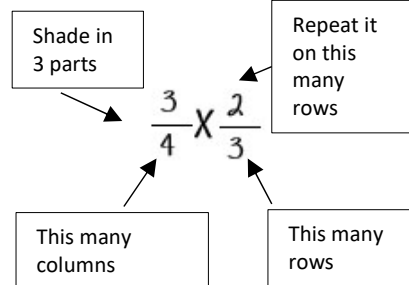
$\frac{4}{5} - \frac{2}{3}$



$= \frac{2}{15}$

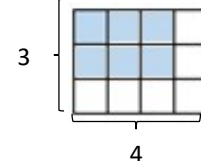
Use equivalent fractions to find a common multiple for both denominators

## Multiplication/ Division of fractions



$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$  Parts shaded

Modelled:



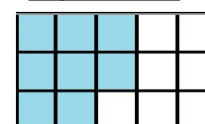
Total number of parts in the diagram

Remember to use reciprocals

$\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times \frac{4}{3}$

Multiplying by a reciprocal gives the same outcome

Represented



$= \frac{8}{15}$

Number Sense

<p><b>Question 1</b> What is <math>2 + (-4)</math>?</p>	<p><b>Question 2</b> Calculate <math>\frac{3}{5} \times \frac{11}{12}</math></p>	<p><b>Question 3</b> Find the HCF of 30 and 40</p>	<p><b>Question 4</b> <span style="float: right;">Give your answer in standard form</span> Calculate <math>(8 \times 10^3) \times (2 \times 10^5)</math></p>
<p><b>Question 5</b> Write this number in correct standard form: <math>0.000314</math></p>	<p><b>Question 6</b> Calculate <math>(-5) - (-7)</math></p>	<p><b>Question 7</b> <span style="float: right;">Give your answer in standard form</span> Calculate <math>(9.2 \times 10^4) - (3.7 \times 10^2)</math></p>	<p><b>Question 8</b> Calculate <math>\frac{1}{4} + \frac{2}{7}</math></p>
<p><b>Question 9</b> Find the LCM of 16 and 24</p>	<p><b>Question 10</b> Calculate <math>\frac{8}{9} - \frac{2}{5}</math></p>	<p><b>Question 11</b> Examples: 2, 20, 100, -7, 63 Non-examples: 21.6, <math>\frac{2}{3}</math>, 82.5, <math>-\frac{6}{7}</math> What are these examples/non-examples of?</p>	<p><b>Question 12</b> Calculate <math>2 \times (-5)</math></p>
<p><b>Question 13</b> Calculate <math>(-7) \times (-4)</math></p>	<p><b>Question 14</b> Calculate <math>20 \div (-4)</math></p>	<p><b>Question 15</b> Calculate <math>(1.4 \times 10^5) \div (7 \times 10^3)</math></p>	<p><b>Question 16</b> Calculate <math>(-4) + (-3)</math></p>
<p><b>Question 17</b> Calculate <math>\frac{2}{3} \div \frac{4}{9}</math></p>	<p><b>Question 18</b> <span style="float: right;">Give your answer in standard form</span> Calculate <math>(2.3 \times 10^5) + (4.7 \times 10^4)</math></p>	<p><b>Question 19</b> Calculate <math>5 - (-4)</math></p>	<p><b>Question 20</b> Write this number in correct standard form: <math>82350000</math></p>

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## What do I need to be able to do?

By the end of this unit you should be able to:

- Find single event probability
- Find relative frequency
- Find expected outcomes
- Find independent events
- Use diagrams to work out probabilities

## Keywords

**Probability:** the chance that something will happen

**Relative Frequency:** how often something happens divided by the outcomes

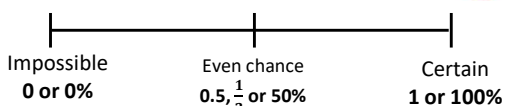
**Independent:** an event that is not effected by any other events.

**Chance:** the likelihood of a particular outcome.

**Event:** the outcome of a probability – a set of possible outcomes.

**Biased:** a built in error that makes all values wrong by a certain amount.

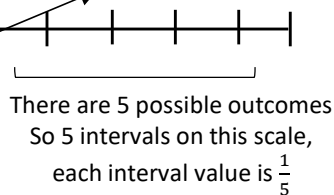
## The probability scale



The more likely an event the further up the probability it will be in comparison to another event (It will have a probability closer to 1)



There are 2 pink and 2 yellow balls, so they have the same probability



## Single event probability

Probability is always a value between 0 and 1



The probability of getting a blue ball is  $\frac{4}{5}$

∴ The probability of **NOT** getting a blue ball is  $\frac{1}{5}$

The sum of the probabilities is 1

The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	

$$P(\text{white chocolate}) = 1 - 0.15 - 0.35 = 0.5$$



## Relative Frequency

$$\frac{\text{Frequency of event}}{\text{Total number of outcomes}}$$

Remember to calculate or identify the overall number of outcomes!

Colour	Frequency	Relative Frequency
Green	6	0.3
Yellow	12	0.6
Blue	2	0.1
	<b>20</b>	

## Expected outcomes

Expected outcomes are estimations. It is a long term average rather than a prediction.

Dark	Milk	White
0.15	0.35	0.5

The sum of the probabilities is 1

An experiment is carried out 400 times.

Show that dark chocolate is expected to be selected 60 times

$$0.15 \times 400 = 60$$

Relative frequency can be used to find expected outcomes

e.g. Use the relative probability to find the expected outcome for green if there are 100 selections.

Relative frequency x Number of times

$$0.3 \times 100 = 30$$

## Independent



The rolling of one dice has no impact on the rolling of the other. The individual probabilities should be calculated separately.

Probability of event 1  $\times$  Probability of event 2

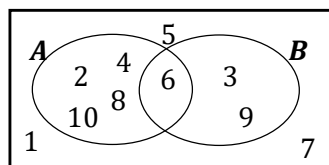


$$P(5) = \frac{1}{6} \quad P(R) = \frac{1}{4}$$

Find the probability of getting a 5 and a red

$$P(5 \text{ and } R) = \frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$$

## Using diagrams



	Car	Bus	Walk	Total
Boys	15	24	14	53
Girls	6	20	21	47
Total	21	44	35	100

The possible outcomes from rolling a dice

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

The possible outcomes from tossing a coin

**Question 1**  
A fair six-sided die is thrown. Mark the probability of each event on the diagram:  
A: A number less than 7 is thrown.  
B: A "6" is thrown.  
C: An odd number is thrown.

**Question 2**  
A fair 4-sided spinner is spun once.

On the probability scale, mark with a letter A, the probability that the spinner will land on the number 4.

**Question 3**  
A bag contains 10 discs. Each disc is labelled with a different number from 1 to 10. A disc is chosen from the bag at random. Write down the probability that the chosen disc is a square number.

**Question 4**  
Elizabeth has a bunch of red, yellow and white roses. She chooses a rose at random.  
The probability that she chooses a yellow rose is 0.1  
The probability that she chooses a white rose is 0.2.  
What is the probability that Elizabeth chooses a rose that is either yellow or white?

**Question 5**  
There are 12 red roses, 5 yellow roses and 3 white roses in a vase.  
Felix takes a rose, at random, from the vase.  
Write down the probability that Felix takes a rose that is not red.

**Question 6**  
A die is rolled 30 times:  
3 4 4 4 6 5 4 5 6 4  
1 3 2 4 4 5 6 4 4 2  
4 5 6 3 2 3 5 6 2 3

Complete the relative frequency table:

Number on dice	1	2	3	4	5	6
Relative frequency						

**Question 7**  
The table shows the probability that a counter taken at random from a bag will be pink, green or blue.

Colour	Pink	Yellow	Green	Blue
Probability	0.5		0.1	0.2

Find the probability of the counter will be yellow.

**Question 8**  
The probability that Ben wins at rugby is 25%.  
If Ben plays rugby 136 times, how many games can Ben expect not to win?

**Question 9**  
Esme takes the bus to university 40 times during a term.  
The relative frequency of the bus being late is 0.3.  
How many times was the bus late?

**Question 10**  
Jack sows 300 wildflower seeds. The probability of a seed flowering is 0.7.  
Work out an estimate for the number of these seeds that will flower.

**Question 11**  
Gary is playing cricket.  
When attempting to catch the ball, the probability Gary is successful is  $\frac{3}{4}$   
During the game, Gary attempts two catches.  
Find the probability Gary is successful with both catches.

**Question 12**  
A fair six sided dice is rolled three times.  
Find the probability of getting no sixes.

**Question 13**  
Harry gets the train to work in the morning. He works Monday to Friday.  
The probability the train is late is 0.2.  
Find the probability the train is late exactly once.

**Question 14**  
Two bags, 1 and 2, each contain three counters. In bag 1, the counters are labelled 1, 2 and 5. In bag 2, the counters are labelled 2, 3 and 4.

Bag 1			
x	1	2	5
2			
3			
4			

Complete the two-way table.

**Question 15**  
Two fair six sided dice are rolled. The numbers on the two dice are added together to give a score.  
Find the probability of scoring a 7.

**Question 16**  
The two-way table shows information about the students in a class:

		Hair colour		
		Brown	Blonde	Red
Glasses	Yes	3	6	3
	No	5	4	1

Find the probability that the student does not wear glasses.

**Question 17**  
80 students visited the library over three days.

	Monday	Tuesday	Wednesday	Total
Year 7			13	38
Year 8	14			
Total		33	26	80

Complete the two way table.

**Question 18**  
Here is a Venn diagram:

Write down  $P(A \cap B)$

**Question 19**  
Here is a Venn Diagram:

Write down the numbers in  $A'$ .

**Question 20**  
 $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$   
A = multiples of 3  
B = multiples of 5  
Complete the Venn diagram.